Attendance

1 - A

MANE 3332.05

Lecture 21

Agenda

- Have worked through partial credit requests
- Continue Chapter 8 lecture
- Chapter 8, Case 1 Practice Problems (assigned 11/6/2025, due 11/11/2025)
- New: Chapter 8, Case 1 Quiz (assigned 11/11/2025, due 11/13/2025)
- New: Chapter 8, Case 2 Practice Problems (assigned 11/11/2025, due 11/13/2025)
- Attendance
- Questions?

Handouts

- Lecture 21 Slides
- Lecture 21 Slides - marked

Week	Tuesday Lecture	Thursday Lecture
11	**11/11** - Chapter 8 (part 2)	**11/13** - Chapter 8 (part 3)
12	**11/18** - Chapter 8 (part 4)	**11/25** - Chapter 9 (part 1)
13	**11/25** - Chapter 9 (part 2)	**11/27** - Thanksgiving Holiday (no class)
14	**12/2** - Chapter 9 (part 3)	**12/4** - Linear Regression
15	**12/9** - Review Session	**12/11** - Study Day (no class)

The final exam for MANE 3332.01 is **Thursday December 18, 2025 at 10:15 AM - 12:00 PM**.

Confidence Interval for the mean of Normal distribution with variance unknown (Case 2)

The t distribution

- Definition.

Let $X_1, X_2, \dots, X_n$ be a random sample from a normal distribution with unknown mean μ and unknown variance σ^2. The random variable

$$T = \frac{\overline{X} - \mu}{S/\sqrt{n}}$$

has a t distribution with $n - 1$ degrees of freedom.

- A table of percentage points (quantiles) is given in Appendix A Table 5
- Figure 8-4 on page 180 shows the relationship between the t and normal distributions.
- Figure 8-5 explains the percentage points of the t distribution

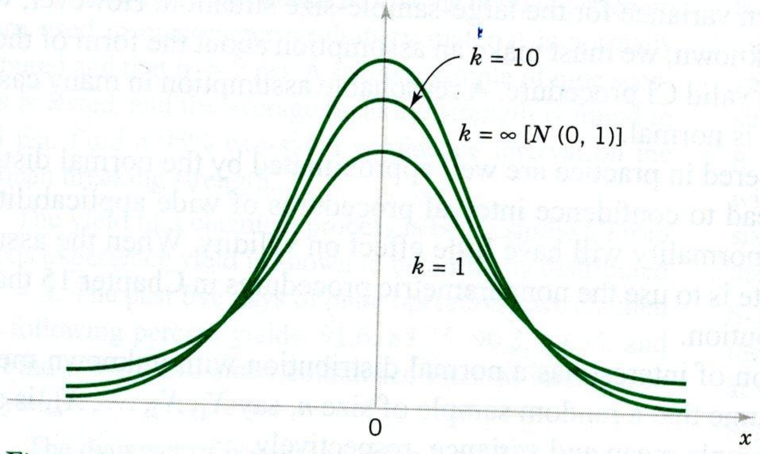

Figure 8-4 Probability density functions of several t distributions.

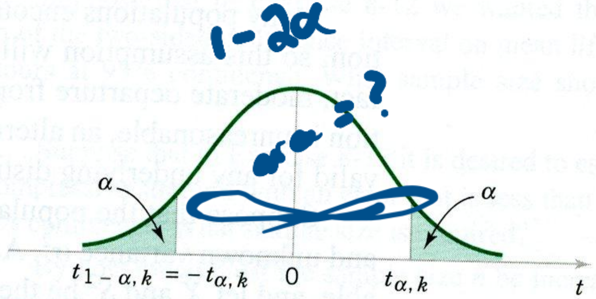

Figure 8-5 Percentage points of the t distribution.

image

Confidence interval definition

- Using the t distribution it is possible to construct CIs

If $\bar{x}$ and s are the mean and standard deviation of a random sample from a normal distribution with unknown variance σ^2, a $100(1-\alpha)\%$ confidence interval on μ is given by

$$\bar{x} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$$

where $t_{\alpha/2,n-1}$ is the upper $100(\alpha/2)$ percentage point of the t distribution with $n-1$ degrees of freedom.

$n = 12$

$\bar{x} = 2.902$

Problem 8-30 (6th edition)

$s = .0999$

$\alpha = 1 - .99 = .01$

8-30. An article in *Nuclear Engineering International* (February 1988, p. 33) describes several characteristics of fuel rods used in a reactor owned by an electric utility in Norway. Measurements on the percentage of enrichment of 12 rods were reported as follows:

2.94	3.00	2.90	2.75	3.00	2.95
2.90	2.75	2.95	2.82	2.81	3.05

(a) Use a normal probability plot to check the normality assumption.

(b) Find a 99% two-sided confidence interval on the mean percentage of enrichment. Are you comfortable with the statement that the mean percentage of enrichment is 2.95 percent? Why?

need $t_{\alpha/2, n-1} = t_{.005, 11} = 3.106$

$\bar{x} - t_{\alpha/2, n-1}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{\alpha/2, n-1}\left(\frac{s}{\sqrt{n}}\right)$

$2.902 - 3.106\left(\frac{.0999}{\sqrt{12}}\right) \le \mu \le 2.902 + 3.106\left(\frac{.0999}{\sqrt{12}}\right)$

Image

$2.8124 \le \mu \le 2.9915$

Table IV Percentage Points $t_{\alpha,\nu}$ of the t-Distribution

ν \ α	.40	.25	.10	.05	.025	.01	.005	.0025	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	.277	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	[illegible]	3.428	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	.255	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	.254	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	.254	.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	.253	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

ν = degrees of freedom.

image

One-sided confidence bounds

- Are easy to construct
- Use only the appropriate upper or lower bound
- Change $t_{\alpha/2,n-1}$ to $t_{\alpha,n-1}$

~~$L \le$~~ $\mu \le u$

upper Bound

$\mu \le \bar{x} + t_{\alpha,n-1}\frac{s}{\sqrt{n}}$

Lower Bound

$\bar{x} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}} \le \mu$

Chapter 8, Case 2 Practice Problems

Question 2 (2 points)

Listen

Consider a sample of size 6 from a normal distribution with mean 18.4 and s 3.06.
What is the value of a two-sided 98.0 % confidence interval for the mean?

- (16.684,20.116)
- ✓ (14.196,22.604)
- (5.537,31.263)
- (14.956,21.844)
- (13.149,23.651)
- (14.098,22.702)

$\alpha = 1 - .98 = .02$

need $t_{\alpha/2, n-1}$

$t_{.02/2, 6-1}$

$t_{.01,5} = 3.365$

$\bar{x} - t_{.01,5}\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{.01,5}\frac{s}{\sqrt{n}}$

$18.4 - 3.365\left(\frac{3.06}{\sqrt{6}}\right) \le \mu \le 18.4 + 3.365\left(\frac{3.06}{\sqrt{6}}\right)$

$14.196 \le \mu \le 22.604$

Question 4 (2 points)

Listen

Consider a sample of size 16 from a normal distribution with mean 24.0 and s 7.13. What is the value of a 97.5 % upper-confidence bound for the mean?

- ( ) mu <=24.95
- ( ) mu <=27.799
- ( ) mu<=30.772
- ( ) mu <=28.438
- ( ) mu<=31.911
- ( ) mu<=51.089

$\alpha = .025$

need $t_{.025, 16-1}$

$t_{.025, 15} = 2.131$

$\mu \le u$

$$\mu \le \bar{x} + t_{\alpha, n-1} \frac{s}{\sqrt{n}}$$

$$\mu \le 24 + 2.131 \frac{7.13}{\sqrt{16}}$$

$$\le 27.7985$$

Question 5 (2 points)

Listen

Consider a sample of size 26 from a normal distribution with mean 127.5 and s 13.2.
What is the value of a 90.0 % lower-confidence bound for the mean?

- mu >=124.092
- mu>=82.519
- mu>=116.053
- mu >=123.078
- mu >=126.832
- mu>=118.678

$\alpha = .1$

$L \le \mu$

need

$t_{.1, 26-1} = 1.316$

$\bar{x} - t_{\alpha, n-1} \frac{s}{\sqrt{n}} \le \mu$

$127.5 - 1.316 \left(\frac{13.2}{\sqrt{26}} \right) \le \mu$

$124.093 \le \mu$

Confidence Interval for σ^2 and σ (Case 3)

- Section 8-3 presents a CI for σ^2 or σ
- Requires the χ^2 (chi-squared) distribution

Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with mean μ and variance σ^2 and let S^2 be the sample variance. Then the random variable

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

has a chi-square (χ^2) distribution with $n-1$ degrees of freedom

- A table of the upper percentage points of the χ^2 distribution are given in Table 4 in the appendix
- Figure 8-9 on page 183 explains the percentage points of the χ^2 distribution

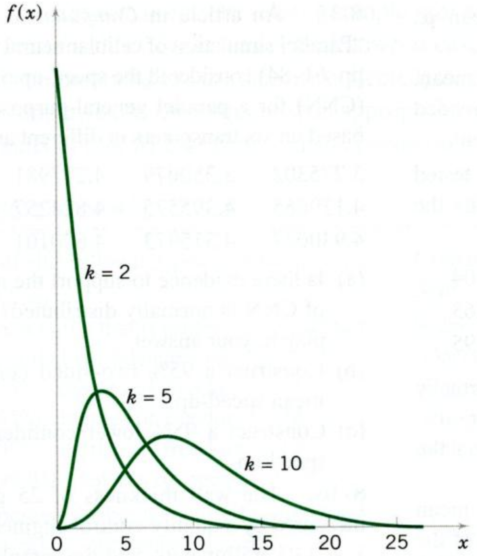

Figure 8-8 Probability density functions of several χ^2 distributions.

image

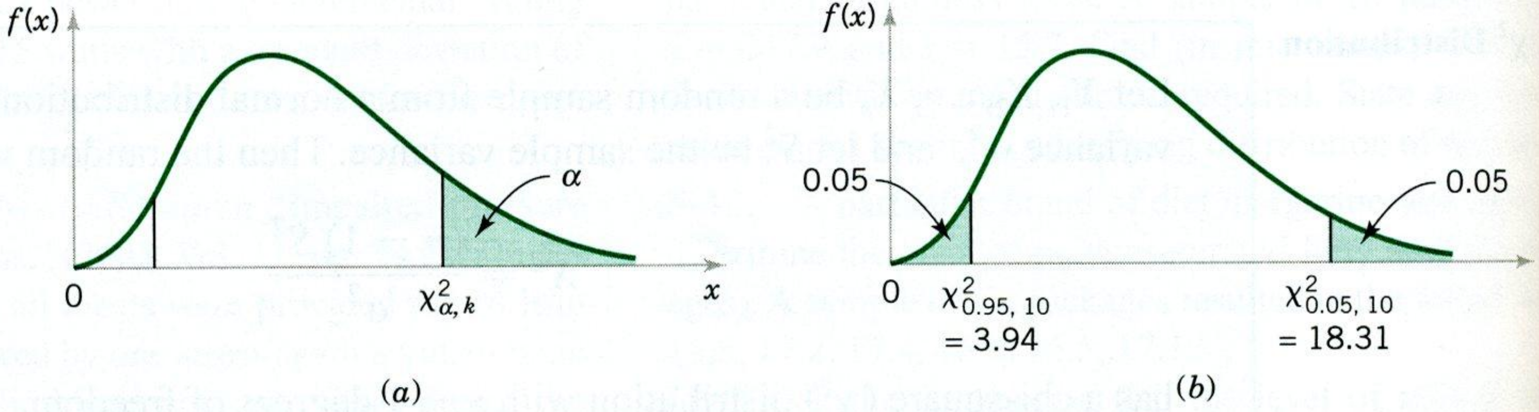

Figure 8-9 Percentage point of the χ^2 distribution. (a) The percentage point $\chi^2_{\alpha,k}$. (b) The upper percentage point $\chi^2_{0.05,10} = 18.31$ and the lower percentage point $\chi^2_{0.95,10} = 3.94$.

image

Confidence Intervals for σ^2 and σ

If s^2 is the sample variance from a random sample of n observations from a normal distribution with unknown variance σ^2, then a $100(1-\alpha)\%$ confidence interval on σ^2 is

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}$$

where $\chi^2_{\alpha/2,n-1}$ and $\chi^2_{1-\alpha/2,n-1}$ are the upper and lower $100\alpha/2$ percentage points of the χ^2-distribution with $n-1$ degrees of freedom

Problem 8-36 (6th edition)

8-36. The sugar content of the syrup in canned peaches is normally distributed. A random sample of $n = 10$ cans yields a sample standard deviation of $s = 4.8$ milligrams. Find a 95% two-sided confidence interval for σ.

image

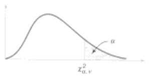

Table III Percentage Points $\chi^2_{\alpha,\nu}$ of the Chi-Squared Distribution

ν \ α	.995	.990	.975	.950	.900	.500	.100	.050	.025	.010	.005
1	.00+	.00+	.00+	.00+	.02	.45	2.71	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	.21	1.39	4.61	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	.58	2.37	6.25	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	2.20	5.35	10.65	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	10.34	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	13.34	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.27	7.26	8.55	14.34	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	15.34	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	16.34	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.87	17.34	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	24.34	34.28	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	25.34	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.65
28	12.46	13.57	15.31	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	59.33	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	69.33	85.53	90.53	95.02	100.42	104.22
80	51.17	53.54	57.15	60.39	64.28	79.33	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	89.33	107.57	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	99.33	118.50	124.34	129.56	135.81	140.17

ν = degrees of freedom.

image

One-sided confidence bounds

- Are easy to construct
- Use only the appropriate upper or lower bound
- Change $\chi^2_{\alpha/2,n-1}$ to $\chi^2_{\alpha,n-1}$ or $\chi^2_{1-\alpha/2,n-1}$ to $\chi^2_{1-\alpha,n-1}$
- See eqn (8-20) on page 184

Chapter 8, Case 3 Practice Problems

Large-Sample CI for a Population Proportion (Case 4)

- Recall from chapter 4, that the sampling distribution of $\hat{P}$ is approximately normal with mean p and variance $p(1-p)/p$, if n is not too close to either 0 or 1 and if n is relatively large.
- Typically, we require both $np \geq 5$ and $n(1-p) \geq 5$

f n is large, the distribution of

$$Z = \frac{X - np}{\sqrt{np(1-p)}} = \frac{\hat{P} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

is approximately standard normal.

If $\hat{p}$ is the proportion of observations in a random sample of size n that belongs to a class of interest, an approximate $100(1-\alpha)\%$ confidence interval on the proportion p of the population that belongs to this class is

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where $z_{\alpha/2}$ is the upper $\alpha/2$ percentage point of the standard normal distribution

Other Considerations

- We can select a sample so that we are $100(1-\alpha)\%$ confident that error $E = |p - \hat{P}|$ using

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p)$$

- An upper bound on is given by

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 (0.25)$$

- One-sided confidence bounds are given in eqn (8-26) on page 187

Guidelines for Constructing Confidence Intervals

- Review excellent guide given in Table 8-1

Other Interval Estimates

- When we want to predict the value of a single value in the future, a **prediction interval** is used
- A **tolerance interval** captures $100(1-\alpha)\%$ of observations from a distribution

Prediction Interval for a Normal Distribution

- Excellent discussion on pages 189 - 190
- A $100(1-\alpha)\%$ PI on a single future observation from a normal distribution is given by

$$\bar{x} - t_{\alpha/2,n-1} s \sqrt{1+\frac{1}{n}} \leq X_{n+1} \leq \bar{x} + t_{\alpha/2,n-1} s \sqrt{1+\frac{1}{n}}$$

Tolerance Intervals for a Normal Distribution

- A **tolerance interval** to contain at least $\gamma\%$ of the values in a normal population with confidence level $100(1-\alpha)\%$ is

$$\bar{x} - ks, \bar{x} + ks$$

where k is a tolerance interval factor for the normal distribution found in appendix A Table XII. Values are given for $1-\alpha$=0.9, 0.95 and 0.99 confidence levels and for $\gamma = .90$, $.95$, and $.99\%$ probability of coverage

- One-sided tolerance bounds can also be computed. The factors are also in Table XII