MANE 3332.05

Lecture 22

Agenda

- Continue Chapter 8 lecture
- Chapter 8, Case 1 Quiz (assigned 11/11/2025, due 11/13/2025)
- Chapter 8, Case 2 Practice Problems (assigned 11/11/2025, due 11/13/2025)
- New: Chapter 8, Case 2 Quiz (assigned 11/13/2025, due 11/18/2025)
- New: Chapter 8, Case 3 Practice Problems (assigned 11/13/2025, due 11/18/2025)
- Attendance
- Questions?

Handouts

- Lecture 22 Slides
- Lecture 22 Slides - marked

Week	Tuesday Lecture	Thursday Lecture
11	**11/11** - Chapter 8 (part 2)	**11/13** - Chapter 8 (part 3)
12	**11/18** - Chapter 8 (part 4)	**11/25** - Chapter 9 (part 1)
13	**11/25** - Chapter 9 (part 2)	**11/27** - Thanksgiving Holiday (no class)
14	**12/2** - Chapter 9 (part 3)	**12/4** - Linear Regression
15	**12/9** - Review Session	**12/11** - Study Day (no class)

The final exam for MANE 3332.01 is **Thursday December 18, 2025 at 10:15 AM - 12:00 PM**.

Summary of One-Sample Hypothesis-Testing Procedures

Case	Null Hypothesis	Test Statistic	Alternative Hypothesis	Fixed Significance Level Criteria for Rejection	P - value	O.C. Curve Parameter	O.C. Curve Appendix Chart VII
1.	$H_0: \mu = \mu_0$ σ^2 known	$z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$H_1: \mu \neq \mu_0$	$\lvert z_0 \rvert > z_{\alpha/2}$	$P = 2[1 - \Phi(\lvert z_0 \rvert)]$	$d = \lvert \mu - \mu_0 \rvert / \sigma$	a, b
			$H_1: \mu > \mu_0$	$z_0 > z_\alpha$	Probability above z_0 $P = 1 - \Phi(z_0)$	$d = (\mu - \mu_0)/\sigma$	c, d
			$H_1: \mu < \mu_0$	$z_0 < -z_\alpha$	Probability below z_0 $P = \Phi(z_0)$	$d = (\mu_0 - \mu)/\sigma$	c, d
2.	$H_0: \mu = \mu_0$ σ^2 unknown	$t_0 = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$H_1: \mu \neq \mu_0$	$\lvert t_0 \rvert > t_{\alpha/2, n-1}$	Sum of the probability above $\lvert t_0 \rvert$ and below $-\lvert t_0 \rvert$	$d = \lvert \mu - \mu_0 \rvert / \sigma$	e, f
			$H_1: \mu > \mu_0$	$t_0 > t_{\alpha, n-1}$	Probability above t_0	$d = (\mu - \mu_0)/\sigma$	g, h
			$H_1: \mu < \mu_0$	$t_0 < -t_{\alpha, n-1}$	Probability below t_0	$d = (\mu_0 - \mu)/\sigma$	g, h
3.	$H_0: \sigma^2 = \sigma_0^2$	$x_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$	$H_1: \sigma^2 \neq \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha/2, n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2, n-1}$	See text Section 9.4.	$\lambda = \sigma/\sigma_0$	i, j
			$H_1: \sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha, n-1}$		$\lambda = \sigma/\sigma_0$	k, l
			$H_1: \sigma^2 < \sigma_0^2$	$\chi_0^2 < \chi^2_{1-\alpha, n-1}$		$\lambda = \sigma/\sigma_0$	m, n
4.	$H_0: p = p_0$	$z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$	$H_1: p \neq p_0$	$\lvert z_0 \rvert > z_{\alpha/2}$	$p = 2[1 - \Phi(\lvert z_0 \rvert)]$	3–4	3–4
			$H_1: p > p_0$	$z_0 > z_\alpha$	Probability above z_0 $p = 1 - \Phi(z_0)$	3–4	3–4
			$H_1: p < p_0$	$z_0 < -z_\alpha$	Probability below z_0 $P = \Phi(z_0)$	3–4	3–4

Summary of One-Sample Confidence Interval Procedures

Case	Problem Type	Point Estimate	Two-sided $100(1-\alpha)$ Percent Confidence Interval
1.	Mean μ, variance σ^2 known	$\bar{x}$	$\bar{x} - z_{\alpha/2}\sigma/\sqrt{n} \le \mu \le \bar{x} + z_{\alpha/2}\sigma/\sqrt{n}$
2.	Mean μ of a normal distribution, variance σ^2 unknown	$\bar{x}$	$\bar{x} - t_{\alpha/2, n-1}s/\sqrt{n} \le \mu \le \bar{x} + t_{\alpha/2, n-1}s/\sqrt{n}$
3.	Variance σ^2 of a normal distribution	s^2	$\dfrac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}} \le \sigma^2 \le \dfrac{(n-1)s^2}{\chi^2_{1-\alpha/2, n-1}}$
4.	Proportion or parameter of a binomial distribution p	$\hat{p}$	$\hat{p} - z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

image

Confidence Interval for σ^2 and σ (Case 3)

- Section 8-3 presents a CI for σ^2 or σ
- Requires the χ^2 (chi-squared) distribution

Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with mean μ and variance σ^2 and let S^2 be the sample variance. Then the random variable

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

has a chi-square (χ^2) distribution with $n-1$ degrees of freedom

- A table of the upper percentage points of the χ^2 distribution are given in Table 4 in the appendix
- Figure 8-9 on page 183 explains the percentage points of the χ^2 distribution

χ - chi

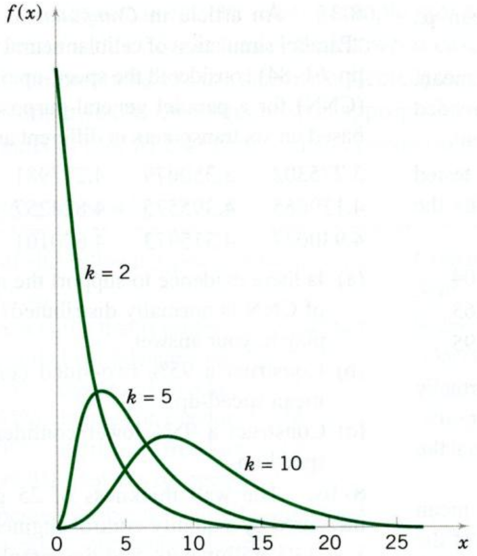

Figure 8-8 Probability density functions of several χ^2 distributions.

image

upper α quantile

90% C.I.

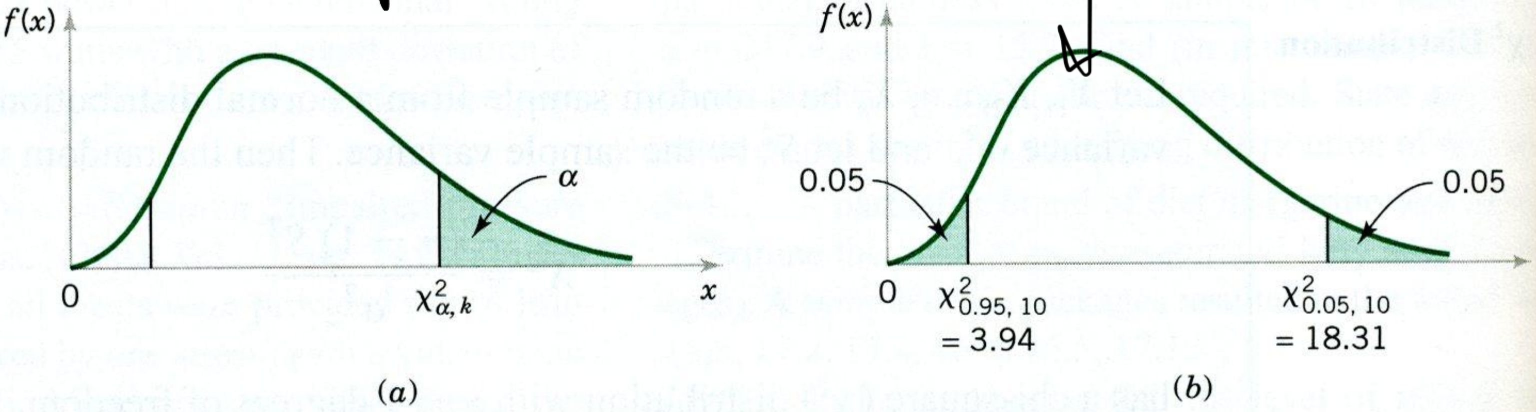

Figure 8-9 Percentage point of the χ^2 distribution. (a) The percentage point $\chi^2_{\alpha,k}$. (b) The upper percentage point $\chi^2_{0.05,10} = 18.31$ and the lower percentage point $\chi^2_{0.95,10} = 3.94$.

image

Case 3

Confidence Intervals for σ^2 and σ

If s^2 is the sample variance from a random sample of n observations from a normal distribution with unknown variance σ^2, then a $100(1-\alpha)\%$ confidence interval on σ^2 is

$$L = \frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}} = U$$

where $\chi^2_{\alpha/2,n-1}$ and $\chi^2_{1-\alpha/2,n-1}$ are the upper and lower $100\alpha/2$ percentage points of the χ^2-distribution with $n-1$ degrees of freedom

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}} \leq \sigma \leq \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}}$$

Problem 8-36 (6th edition)

8-36. The sugar content of the syrup in canned peaches is normally distributed. A random sample of $n = 10$ cans yields a sample standard deviation of $s = 4.8$ milligrams. Find a 95% two-sided confidence interval for σ.

σ or σ^2

$n = 10$

$s = 4.8$

$\alpha = .05$

need $\chi^2_{\alpha/2, n-1} = \chi^2_{.025, 9} = 19.02$

$\chi^2_{1-\alpha/2, n-1} = \chi^2_{.975, 9} = 2.7$

$$\frac{(n-1)s^2}{\chi^2_{.025, 9}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{.975, 9}}$$

$$10.902 = \frac{(10-1)(4.8)^2}{19.02} \le \sigma^2 \le \frac{(10-1)(4.8)^2}{2.7} = 76.8$$

$$\sqrt{10.902} = 3.302 < \sigma < \sqrt{76.8} = 8.764$$

image

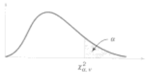

Table III Percentage Points $\chi^2_{\alpha,\nu}$ of the Chi-Squared Distribution

ν \ α	.995	.990	.975	.950	.900	.500	.100	.050	.025	.010	.005
1	.00+	.00+	.00+	.00+	.02	.45	2.71	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	.21	1.39	4.61	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	.58	2.37	6.25	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	2.20	5.35	10.65	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	9.34	15.99	18.31	[illegible]	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	10.34	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	13.34	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.27	7.26	8.55	14.34	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	15.34	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	16.34	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.87	17.34	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	24.34	34.28	[illegible]	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	[illegible]	25.34	[illegible]	[illegible]	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.65
28	12.46	13.57	15.31	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	59.33	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	69.33	85.53	90.53	95.02	100.42	104.22
80	51.17	53.54	57.15	60.39	64.28	79.33	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	89.33	107.57	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	99.33	118.50	124.34	129.56	135.81	140.17

ν = degrees of freedom.

image

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} = L \le \sigma^2 \le U = \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}$$

One-sided confidence bounds

- Are easy to construct
- Use only the appropriate upper or lower bound
- Change $\chi^2_{\alpha/2,n-1}$ to $\chi^2_{\alpha,n-1}$ or $\chi^2_{1-\alpha/2,n-1}$ to $\chi^2_{1-\alpha,n-1}$
- See eqn (8-20) on page 184

Lower Bond

$L \le \sigma^2 \cancel{\le U}$

$$\frac{(n-1)s^2}{\chi^2_{\alpha,n-1}} \le \sigma^2$$

Upper Bond

$$\sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\alpha,n-1}}$$

Chapter 8, Case 3 Practice Problems

Question 2 (2 points)

$1-\alpha/2 = 1-.2/2 = 1-.1 = .9$

Consider a sample of size 26 from a normal distribution with mean 127.1 and sample standard deviation 25.47. What is the value of a two-sided 80.0 % confidence interval for the variance?

s

$\alpha = .2$

σ^2

- (528.619,856.284)
- (471.728,984.701)
- (456.075,938.0)
- (120.525,133.675)
- (21.719,31.38)

$34.28 = \chi^2_{\alpha/2, n-1} = \chi^2_{.1, 25}$

$16.47 = \chi^2_{1-\alpha/2, n-1} = \chi^2_{.9, 25}$

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\alpha/2, n-1}}$$

$$\frac{(26-1)(25.47)^2}{34.28} = 473.105 \le \sigma^2 \le \frac{(26-1)(25.47)^2}{16.47} = 984.70$$

Question 4 (2 points)

Listen

Consider a sample of size 12 from a normal distribution with mean 79.5 and sample standard deviation 17.11. What is the value of lower 97.5 % confidence bound on the variance?

- 133.843
- 146.91
- 137.972
- 12.121
- 68.629

$\alpha = .025$

$\chi^2_{.025, 11} = 21.92$

$L \le \sigma^2$

$$\frac{(n-1)s^2}{\chi^2_{\alpha, n-1}} \le \sigma^2$$

$$\frac{(12-1)(17.11)^2}{21.92} \le \sigma^2$$

$146.91 \le \sigma^2$

Question 5 (2 points)

Listen

Consider a sample of size 18 from a normal distribution with mean 92.2 and sample standard deviation 21.54. What is the value of an upper 95.0 % confidence bound on the standard deviation?

σ

$\alpha = .05$

- 32.301
- 101.032
- ~~30.162~~
- 28.983
- 909.748

$\chi^2_{.95,17} = 8.67$

$$\sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha, n-1}}$$

$$\sigma^2 \leq \frac{(18-1)(21.54)^2}{8.67} = 909.748$$

$$\sigma \leq \sqrt{909.748} = 30.162$$

Large-Sample CI for a Population Proportion (Case 4)

- Recall from chapter 4, that the sampling distribution of $\hat{P}$ is approximately normal with mean p and variance $p(1-p)/p$, if n is not too close to either 0 or 1 and if n is relatively large.
- Typically, we require both $np \geq 5$ and $n(1-p) \geq 5$

f n is large, the distribution of

$$Z = \frac{X - np}{\sqrt{np(1-p)}} = \frac{\hat{P} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

is approximately standard normal.

totals

proportion

If $\hat{p}$ is the proportion of observations in a random sample of size n that belongs to a class of interest, an approximate $100(1 - \alpha)\%$ confidence interval on the proportion p of the population that belongs to this class is

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where $z_{\alpha/2}$ is the upper $\alpha/2$ percentage point of the standard normal distribution

Other Considerations

- We can select a sample so that we are $100(1-\alpha)\%$ confident that error $E = |p - \hat{P}|$ using

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p)$$

→ round up

- An upper bound on is given by

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 (0.25)$$

- One-sided confidence bounds are given in eqn (8-26) on page 187

Guidelines for Constructing Confidence Intervals

- Review excellent guide given in Table 8-1

Other Interval Estimates

- When we want to predict the value of a single value in the future, a **prediction interval** is used
- A **tolerance interval** captures $100(1-\alpha)\%$ of observations from a distribution

Prediction Interval for a Normal Distribution

- Excellent discussion on pages 189 - 190
- A $100(1-\alpha)\%$ PI on a single future observation from a normal distribution is given by

$$\bar{x} - t_{\alpha/2,n-1} s \sqrt{1 + \frac{1}{n}} \leq X_{n+1} \leq \bar{x} + t_{\alpha/2,n-1} s \sqrt{1 + \frac{1}{n}}$$

Tolerance Intervals for a Normal Distribution

- A **tolerance interval** to contain at least $\gamma\%$ of the values in a normal population with confidence level $100(1-\alpha)\%$ is

$$\bar{x} - ks, \bar{x} + ks$$

where k is a tolerance interval factor for the normal distribution found in appendix A Table XII. Values are given for $1-\alpha$=0.9, 0.95 and 0.99 confidence levels and for $\gamma = .90$, $.95$, and $.99\%$ probability of coverage

- One-sided tolerance bounds can also be computed. The factors are also in Table XII