Attendance: 1 - D

MANE 3332.05

Lecture 24

Agenda

- Continue Chapter 9 lecture
  - Resume with Connection to confidence intervals and p-values for Case 1
- Chapter 8, Case 3 Quiz (assigned 11/18/2025, due 11/20/2025)
- NEW: Chapter 9, Case 1 2-sided Practice Problems (assigned 11/20/2025, due 11/25/2025)
- NEW: Chapter 9, Case 1 Lower Practice Problems (assigned 11/20/2025, due 11/25/2025)
- NEW: Chapter 9, Case 1 Upper Practice Problems (assigned 11/20/2025, due 11/25/2025)
- Attendance
- Questions?

Handouts

- Lecture 24 Slides
- Lecture 24 Slides - marked

Week	Tuesday Lecture	Thursday Lecture
12	**11/18** - Chapter 9 (part 1)	**11/25** - Chapter 9, Case 1
13	**11/25** - Chapter 9, Case 2	**11/27** - Thanksgiving Holiday (no class)
14	**12/2** - Chapter 9, Case 3	**12/4** - Linear Regression
15	**12/9** - Review Session	**12/11** - Study Day (no class)

The final exam for MANE 3332.05 is **Thursday December 18, 2025 at 1:15 - 3:00 PM**.

Summary of One-Sample Hypothesis-Testing Procedures

Case	Null Hypothesis	Test Statistic	Alternative Hypothesis	Fixed Significance Level Criteria for Rejection	P - value	O.C. Curve Parameter	O.C. Curve Appendix Chart VII
1.	$H_0: \mu = \mu_0$ σ^2 known	$z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$H_1: \mu \neq \mu_0$	$\lvert z_0 \rvert > z_{\alpha/2}$	$P = 2[1 - \Phi(\lvert z_0 \rvert)]$	$d = \lvert \mu - \mu_0 \rvert / \sigma$	a, b
			$H_1: \mu > \mu_0$	$z_0 > z_\alpha$	Probability above z_0 $P = 1 - \Phi(z_0)$	$d = (\mu - \mu_0)/\sigma$	c, d
			$H_1: \mu < \mu_0$	$z_0 < -z_\alpha$	Probability below z_0 $P = \Phi(z_0)$	$d = (\mu_0 - \mu)/\sigma$	c, d
2.	$H_0: \mu = \mu_0$ σ^2 unknown	$t_0 = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$H_1: \mu \neq \mu_0$	$\lvert t_0 \rvert > t_{\alpha/2, n-1}$	Sum of the probability above $\lvert t_0 \rvert$ and below $-\lvert t_0 \rvert$	$d = \lvert \mu - \mu_0 \rvert / \sigma$	e, f
			$H_1: \mu > \mu_0$	$t_0 > t_{\alpha, n-1}$	Probability above t_0	$d = (\mu - \mu_0)/\sigma$	g, h
			$H_1: \mu < \mu_0$	$t_0 < -t_{\alpha, n-1}$	Probability below t_0	$d = (\mu_0 - \mu)/\sigma$	g, h
3.	$H_0: \sigma^2 = \sigma_0^2$	$x_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$	$H_1: \sigma^2 \neq \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha/2, n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2, n-1}$	See text Section 9.4.	$\lambda = \sigma/\sigma_0$	i, j
			$H_1: \sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha, n-1}$		$\lambda = \sigma/\sigma_0$	k, l
			$H_1: \sigma^2 < \sigma_0^2$	$\chi_0^2 < \chi^2_{1-\alpha, n-1}$		$\lambda = \sigma/\sigma_0$	m, n
4.	$H_0: p = p_0$	$z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$	$H_1: p \neq p_0$	$\lvert z_0 \rvert > z_{\alpha/2}$	$p = 2[1 - \Phi(\lvert z_0 \rvert)]$	3–4	3–4
			$H_1: p > p_0$	$z_0 > z_\alpha$	Probability above z_0 $p = 1 - \Phi(z_0)$	3–4	3–4
			$H_1: p < p_0$	$z_0 < -z_\alpha$	Probability below z_0 $P = \Phi(z_0)$	3–4	3–4

Summary of One-Sample Confidence Interval Procedures

Case	Problem Type	Point Estimate	Two-sided $100(1-\alpha)$ Percent Confidence Interval
1.	Mean μ, variance σ^2 known	$\bar{x}$	$\bar{x} - z_{\alpha/2}\sigma/\sqrt{n} \le \mu \le \bar{x} + z_{\alpha/2}\sigma/\sqrt{n}$
2.	Mean μ of a normal distribution, variance σ^2 unknown	$\bar{x}$	$\bar{x} - t_{\alpha/2, n-1}\, s/\sqrt{n} \le \mu \le \bar{x} + t_{\alpha/2, n-1} s/\sqrt{n}$
3.	Variance σ^2 of a normal distribution	s^2	$\dfrac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}} \le \sigma^2 \le \dfrac{(n-1)s^2}{\chi^2_{1-\alpha/2, n-1}}$
4.	Proportion or parameter of a binomial distribution p	$\hat{p}$	$\hat{p} - z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

Introduction to Hypothesis Testing

Decision Making for a Single Sample

- Inferential statistics consists of methods used to make decisions or draw conclusions about a population using information contained in a sample
- Inference is divided into two major areas:
  - Parameter estimation (both point and interval)
  - Hypothesis testing

Overview of Statistical Hypotheses

- Many engineering problems require a decision to be made regarding some statement about a parameter
  - The statement is called a **hypothesis**
  - The decision-making process about the hypothesis is call **hypothesis testing**
- Statistical hypothesis testing is usually the data analysis stage of a **comparative experiment**
- A procedure leading to a decision about a particular hypothesis is called a **test of hypothesis**
- Testing the hypothesis involves taking a random sample, computing a **test statistic** from the sample data and then using the to make a decision

Statistical Hypothesis

- A **statistical hypothesis** is a statement about the parameters of one or more populations
- A statistical hypothesis has two parts a null hypothesis (denoted H_0) and an alternative hypothesis (denoted H_1)
  - The null hypothesis contains an equality statement about the value of parameter. For example $H_0: \mu = 12$ ounces.
  - There are three possible alternative hypotheses: $H_1: \mu \neq 12$, $H_1: \mu < 12$, or $H_1: \mu > 12$
  - The goal of the research will determine the appropriate alternative hypothesis

Summary of One-Sample Hypothesis-Testing Procedures

Case	Null Hypothesis	Test Statistic	Alternative Hypothesis
1.	$H_0: \mu = \mu_0$ σ^2 known	$z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$H_1: \mu \neq \mu_0$ $H_1: \mu > \mu_0$ $H_1: \mu < \mu_0$

Errors in hypothesis testing

Whether a correct decision is made depends upon the true nature of H_0 and the decision arrived at.

- A **type I error** occurs when the null hypothesis is true and the outcome of the test is to reject H_0. The probability of a type I error is denoted as α
- A **type II error** occurs when the null hypothesis is false and the outcome of the test is to fail to reject H_0. The probability of a type II error is denoted as β.
- The **power** of a statistical test is the probability rejecting the null hypothesis H_0 when the alternative hypothesis is true. Power $= 1 - \beta$

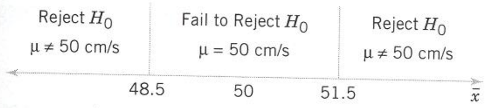


Figure 9-1 Decision criteria for testing H_0: $\mu = 50$ centimeters per second versus H_1: $\mu \neq 50$ centimeters per second.

Table 9-1 Decisions in Hypothesis Testing

Decision	H_0 Is True	H_0 Is False
Fail to reject H_0	no error	type II error
Reject H_0	type I error	no error

Error Example: Manufacturing

Error Example: Medical

General Procedure for Hypothesis Testing

The following sequence of steps is recommended

1. From the problem context, identify the parameter of interest,
2. State the null hypothesis, H_0,
3. Specify an appropriate alternative hypothesis, H_1,
4. Choose a significance level α
5. State an appropriate test statistic,
6. State the rejection region for the (test) statistic,
7. Compute any necessary sample quantities, substitute these into the equation for the test statistics, and compute that value,
8. Decide whether or not H_0 should be rejected and report in the problem context

Chapter 9, Case 1

Inference on the Mean of a population, variance known

- Assumptions:
  1. $X_1, X_2, \ldots, X_n$ is a random sample of size n from a population
  2. The population is normal, or if it is not normal, the conditions of the central limit theorem apply
- The parameter of interest is μ
- The null hypothesis is $H_0: \mu = \mu_0$
- The test statistic is

$$Z_0 = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}$$

and has a standard normal distribution

The alternative hypotheses and corresponding critical value(s) are shown in figure 9-11 on page 209

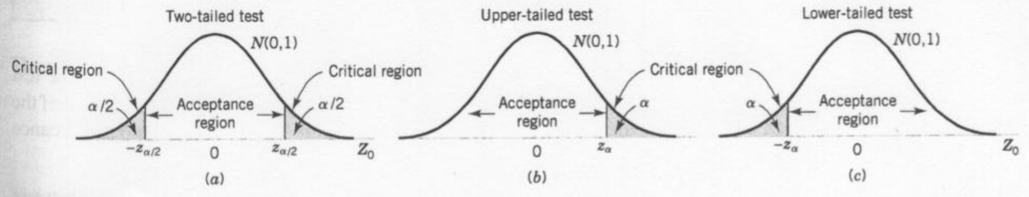


FIGURE 9.11

The distribution of Z_0 when H_0: $\mu = \mu_0$ is true with critical region for (a) the two-sided alternative H_1: $\mu \neq \mu_0$, (b) the one-sided alternative H_1: $\mu > \mu_0$, and (c) the one-sided alternative H_1: $\mu < \mu_0$.

Summary for hypothesis test on the mean, variance known

See the material on the inside cover of your textbook

mmary of One-Sample Hypothesis-Testing Procedures

Case	Null Hypothesis	Test Statistic	Alternative Hypothesis	Fixed Significance Level Criteria for Rejection	*P*-Value	O.C. Curve Parameter	O.C. Curve Appendix Chart VII
.	$H_0: \mu = \mu_0$ σ^2 known	$z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$H_1: \mu \neq \mu_0$	$\lvert z_0 \rvert > z_{\alpha/2}$	$P = 2[1 - \Phi(z_0)]$	$d = \lvert \mu - \mu_0 \rvert / \sigma$	a, b
			$H_1: \mu > \mu_0$	$z_0 > z_\alpha$	Probability above z_0 $P = 1 - \Phi(z_0)$	$d = (\mu - \mu_0)/\sigma$	c, d
			$H_1: \mu < \mu_0$	$z_0 < -z_\alpha$	Probability below z_0 $P = \Phi(z_0)$	$d = (\mu_0 - \mu)/\sigma$	c, d

Problem 1

Example 11-1

The burning rate of a rocket propellant is being studied. Specifications require that the mean burning rate must be 40 cm/s. Furthermore, suppose that we know that the standard deviation of the burning rate is approximately 2 cm/s. The experimenter decides to specify a type I error probability $\alpha = 0.05$, and he will base the test on a random sample of size $n = 25$. The hypotheses we wish to test are

$$H_0: \mu = 40 \text{ cm/s},$$
$$H_1: \mu \neq 40 \text{ cm/s}.$$

Twenty-five specimens are tested, and the sample mean burning rate obtained is $\bar{x} = 41.25$ cm/s.

Source: Hines, Montgomery, Goldsman, Borror (2003). Probability and Statistics in Engineering, 4th ed.

Problem 2

9.2.10 The bacterial strain *Acinetobacter* has been tested for its adhesion properties. A sample of five measurements gave readings of 2.69, 5.76, 2.67, 1.62 and 4.12 dyne-cm^2. Assume that the standard deviation is known to be 0.66 dyne-cm^2 and that the scientists are interested in high adhesion (at least 2.5 dyne-cm^2).

a. Should the alternative hypothesis be one-sided or two-sided?

b. Test the hypothesis that the mean adhesion is 2.5 dyne-cm^2.

c. What is the *P*-value of the test statistic?

Summary Statistics

```
x<-c(2.69,5.76,2.67,1.26,4.12)
library(psych)
describe(x)

##    vars n mean   sd median trimmed  mad  min  max range skew kurtosis   se
## X1    1 5  3.3 1.71   2.69     3.3 2.12 1.26 5.76   4.5 0.26    -1.71 0.76
```

Connection between Hypothesis Tests and CI

- There is a close connection between confidence intervals and hypothesis tests
- Consider a $100(1-\alpha)\%$ confidence interval on μ and a hypothesis test of size α shown below

$$H_0: \mu = \mu_0$$
$$H_1: \mu \neq \mu_0$$

- The conclusion to reject H_0 will be reached if μ_0 is not contained within the confidence interval
- If μ_0 is within the confidence interval, we fail to reject H_0
- The $100(1-\alpha)\%$ confidence interval on μ is the acceptance region

P-values

- Is a widely used alternative to the traditional hypothesis test
- Definition: The p-value is the smallest level of significance that would lead to reject of the null hypothesis H_0 with the given data
- Formulas are given below

$$P = \begin{cases} 2[1 - \Phi(|z_0|)] & \text{for a two-tailed test} \\ 1 - \Phi(z_0) & \text{for a upper-tailed test} \\ \Phi(z_0) & \text{for a lower-tailed test} \end{cases}$$

- Usage: if p-value<α then the conclusion is reject H0, otherwise fail to reject H0

Test 1

$z_0 = 1.95$, Reject if $|z_0| > 1.96$

fail to reject

Test 2

$z_0 = 1.97$, Reject H_0 if $|z_0| > 1.96$

Reject H_0

Classical Approach (fixed rejection region)

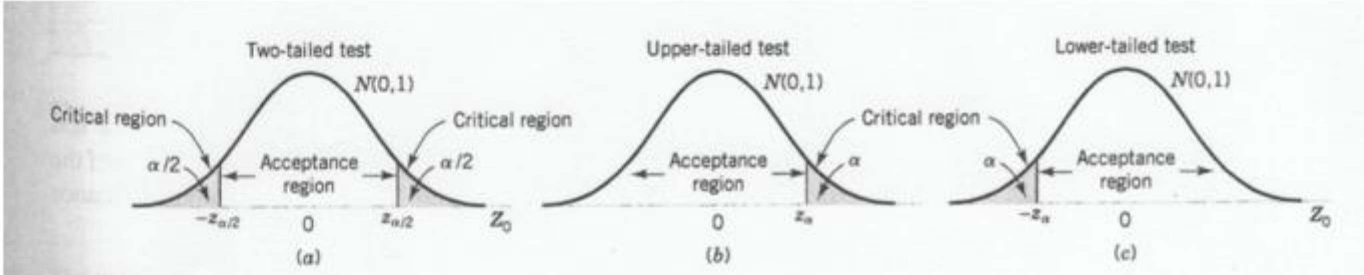
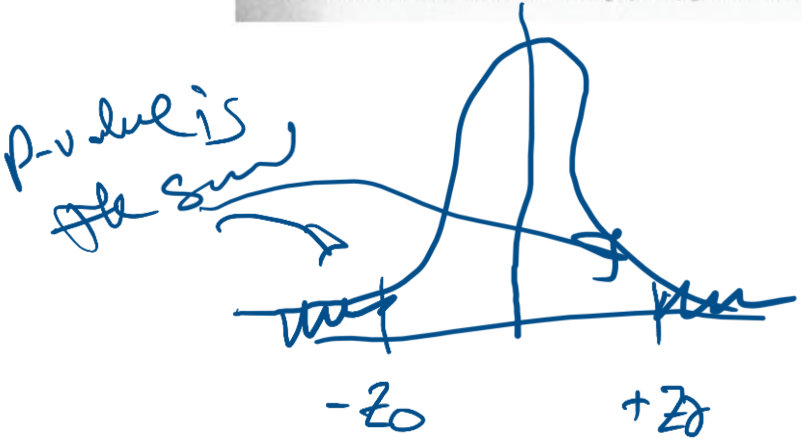
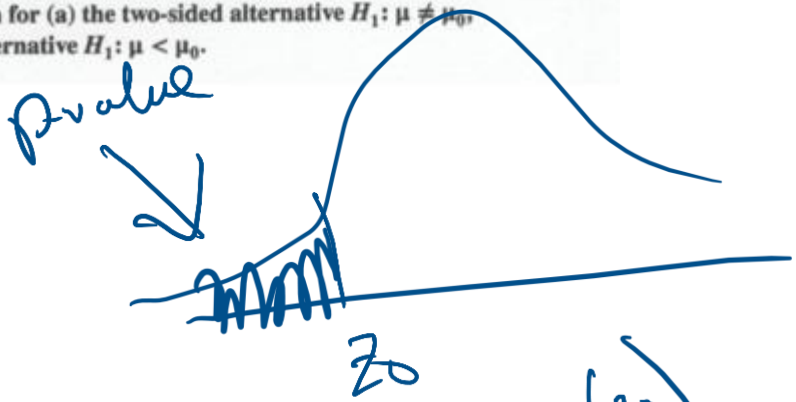


FIGURE 9.11

The distribution of Z_0 when H_0: $\mu = \mu_0$ is true with critical region for (a) the two-sided alternative H_1: $\mu \neq \mu_0$, (b) the one-sided alternative H_1: $\mu > \mu_0$, and (c) the one-sided alternative H_1: $\mu < \mu_0$.





p-value = $\Phi(z_0)$

p-value from Problem 1 on Tuesday

$H_1: \mu \neq \mu_0$

formula for p-value

$$p\text{-value} = 2\left[1 - \Phi(|z_0|)\right]$$

$$= 2\left[1 - \Phi(|3.125|)\right]$$

$$= 2\left[1 - \Phi(3.13)\right]$$

$$= 2\left[1 - .99126\right] = 2(.000874) = .001748$$

$\alpha = .05$. Since p-value $< \alpha$, we reject H_0

Practice Problem Chapter 9, Case 1 2-sided

Question 1 (2 points)

Listen

$H_0: \mu = 18.5$

$H_1: \mu \neq 18.5$

Calculate the test statistic for a test of hypothesis for the mean of single sample with variance known. The null hypothesis is mu=18.5 versus mu not equal to 18.5 using alpha=0.002. The sample statistics are n=21, xbar=19.78, sigma=2.871.

- [x] z0=2.0431
- [ ] z0=9.3626
- [ ] z0=-2.0431
- [ ] z0=3.2611

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{19.78 - 18.5}{2.871/\sqrt{21}} = 2.04308$$

Question 2 (2 points)

Listen

Construct the rejection region for a test of hypothesis for the mean of single sample with variance known. The null hypothesis is mu=25.8 versus mu not equal to 25.8 using alpha=0.002. The sample statistics are n=6, xbar=30.77, sigma=6.579.

- Reject Ho if z0>2.878
- Reject Ho if Z0<-2.878
- Reject Ho if z0>3.09
- Reject H0 if |z0|>3.09
- Reject Ho if Z0<-3.09
- Reject H0 if |z0|>2.878

$\neq$

Reject Ho if $|Z_0| > Z_{\alpha/2}$

$= Z_{.002/2}$

$= Z_{.001}$

$= 3.090$

Question 3 (2 points)

Listen

Which is the correct conclusion for a test of hypothesis for the mean of single sample with variance known. The null hypothesis is mu=0.5 versus mu not equal to 0.5 using alpha=0.02. The sample statistics are n=10, xbar=0.48, sigma=0.032. The value of Z0 is -1.9764 and the rejection region is reject HO if |z0|>2.326

- ~~Reject H0~~
- Fail to reject H0

$Z_0 = -1.9764$, RR: reject H_0 if $|Z_0| > 2.326$

is $|-1.9764| > 2.326$?

No, so fail to reject H_0

Question 4 (2 points)

Listen

Find the p-value for a test of hypothesis for the mean of single sample with variance known. The null hypothesis is mu=12.0 versus mu not equal to 12.0 using alpha=0.02. The sample statistics are n=8, xbar=13.26, sigma=2.289 and the value of the test statistic, Z0, is 1.5569.

- p-value=0.05938
- p-value=0.94062
- ✓ p-value=0.11876

p-value → linked to $H_1: \mu \neq 12$

$$\text{p-value} = 2[1 - \Phi(|Z_0|)]$$
$$= 2[1 - \Phi(1.56)]$$
$$= 2\{1 - .94062\}$$
$$= 2[.05938] = .11876$$

Question 5 (2 points)

Listen

Using the p-value from a test of hypothesis for the mean of single sample with variance known, determine the correct conclusion for the hypothesis test. The null hypothesis is mu=32.0 versus mu not equal to 32.0 using alpha=0.2. The sample statistics are n=29, xbar=32.82, sigma=4.046. The results of hypothesis test include z0= 1.0914 and p-value=0.275097.

- ○ Reject H0
- ○ ✓ Fail to reject H0

Q: is p-value < α

.275097 < .2

? no, we fail to reject Ho

Practice Problem Chapter 9, Case 1 Lower

Question 2 (2 points)

Listen

Construct the rejection region for a test of hypothesis for the mean of single sample with variance known. The null hypothesis is mu=12.0 versus mu less than 12.0 using alpha=5.0E-4. The sample statistics are n=14, xbar=10.39, sigma=2.047.

- Reject Ho if |z0|>3.481
- Reject Ho if Z0<-3.481
- Reject H0 if z0>3.291
- Reject H0 if z0<-3.291

Question 4 (2 points)

p-value is tied to $H_1: \mu < 18.5$

Listen

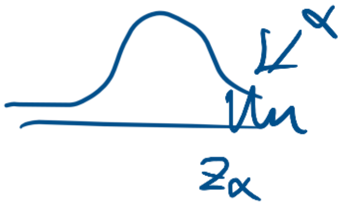
Find the p-value for a test of hypothesis for the mean of single sample with variance known. The null hypothesis is mu=18.5 versus mu less than 18.5 using alpha=0.025. The sample statistics are n=5, xbar=15.45, sigma=2.657 and the value of the test statistic, Z0, is -2.5668.

- ○ p-value=0.01017
- ○ p-value=0.994915
- ✓ p-value=0.005085

$$\text{p-value} = \Phi(Z_0) = \Phi(-2.57) = .005085$$

Practice Problem Chapter 9, Case 1 Upper

$H_1: \mu > 25.8$



Question 2 (2 points)

Listen

Construct the rejection region for a test of hypothesis for the mean of single sample with variance known. The null hypothesis is mu=25.8 versus mu greater than 25.8 using alpha=0.01. The sample statistics are n=15, xbar=23.84, sigma=4.739.

Reject Ho if $Z_0 > Z_\alpha = Z_{.01}$
$= 2.326$

- Reject H0 if z0<-2.326
- Reject Ho if |z0|>2.576
- ✓ Reject H0 if z0>2.326

Question 4 (2 points)

Listen

Find the p-value for a test of hypothesis for the mean of single sample with variance known. The null hypothesis is mu=0.5 versus mu greater than 0.5 using alpha=0.025. The sample statistics are n=15, xbar=0.51, sigma=0.048 and the value of the test statistic, Z0, is 0.8069.

- p-value=0.20897
- p-value=0.41794
- p-value=0.79103

$$p\text{-value} = 1 - \Phi(z_0)$$
$$= 1 - \Phi(.81)$$
$$= 1 - .79103$$
$$= .20897$$

Do not add to note card

Type II error and sample size for a two-tailed test

- Probability of type II error for the two-tailed test

$$\beta = \Phi\left(z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right)$$

where $\mu = \mu_0 + \delta$

- The sample to detect a difference between the true and hypothesized mean of δ with power at least $1 - \beta$ is

$$n \approx \frac{(z_{\alpha/2} + z_\beta)^2 \sigma^2}{\delta^2}$$

where $\delta = \mu - \mu_0$

Type II error and sample size for the one-tailed tests

- For an upper-tailed test

$$\beta = \Phi\left(z_\alpha - \frac{\delta\sqrt{n}}{\sigma}\right)$$

- For a lower-tailed test

$$\beta = 1 - \Phi\left(-z_\alpha - \frac{\delta\sqrt{n}}{\sigma}\right)$$

- The sample size required to detect a difference between the true mean and hypothesized mean of δ with power at least $1 - \beta$ is

$$n = \frac{(z_\alpha + z_\beta)^2\sigma^2}{\delta^2}$$

If n is not an integer, round up to the nearest integer

R: Chapter 9 Case 1 Hypothesis Testing

Z-test

```
library(BSDA)

## Loading required package: lattice

##
## Attaching package: 'BSDA'

## The following object is masked from 'package:datasets':
##
##     Orange

z.test(x,alternative='greater',mu=2.5,sigma.x=0.66,conf.level=0.95)

##
##  One-sample z-Test
##
## data:  x
## z = 2.7104, p-value = 0.00336
## alternative hypothesis: true mean is greater than 2.5
## 95 percent confidence interval:
##  2.814503       NA
## sample estimates:
## mean of x
##       3.3
```

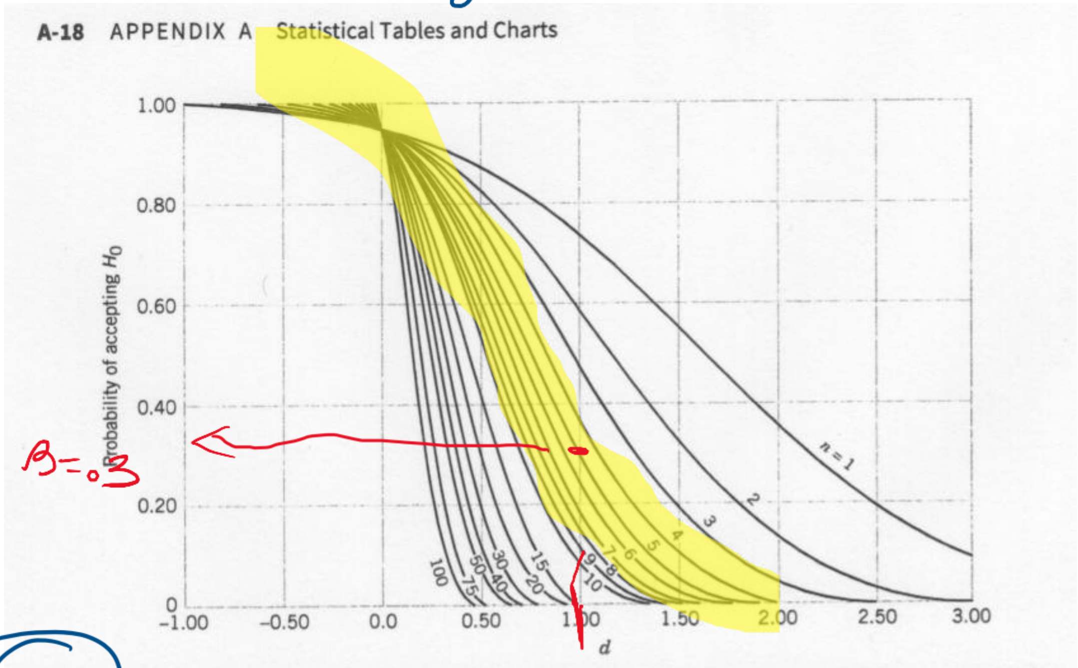
Pretend $d = 1.0$

$n = 5$

$d = \frac{\mu - \mu_0}{\sigma}$

Power using OC-Curve

Find the power when the true mean value is 3.325



(c) O.C. curves for different values of n for the one-sided normal test for a level of significance $\alpha = 0.05$.

Table C

$\alpha = .05$ $d = 1$

R: Chapter 9 Case 1 Power

Power

```
library(asbio)

## Warning: package 'asbio' was built under R version 4.2.3

## Loading required package: tcltk

##
## Attaching package: 'asbio'

## The following object is masked from 'package:psych':
##
##     skew

power.z.test(sigma=0.66,n=5,alpha=0.05,effect=0.825,test="one.tail")

## $sigma
## [1] 0.66
##
## $n
## [1] 5
##
## $power
## [1] 0.8749757
##
## $alpha
## [1] 0.05
##
## $effect
## [1] 0.825
##
## $test
## [1] "one.tail"
```

← Power $= 1-\beta$

Type II Error Rate and Sample Size

- You will not be required to calculate or use OC-curves
- You must understand the concept and be able to correctly identity type I and type II error

Chapter 9, Case 2

Hypothesis Test on the Mean, Variance Unknown

- Much more common case than variance known
- Substitute S for σ
- The test statistics is now a t random variable

$$T = \frac{\overline{X} - \mu_0}{S/\sqrt{n}}$$

Summary of Case 2

$H_0: \mu = \mu_0$ σ^2 unknown	$t_0 = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$H_1: \mu \neq \mu_0$	$\lvert t_0 \rvert > t_{\alpha/2, n-1}$	Sum of the probability above $\lvert t_0 \rvert$ and below $-\lvert t_0 \rvert$	$d = \lvert \mu - \mu_0 \rvert / \sigma$	e, f
		$H_1: \mu > \mu_0$	$t_0 > t_{\alpha, n-1}$	Probability above t_0	$d = (\mu - \mu_0)/\sigma$	g, h
		$H_1: \mu < \mu_0$	$t_0 < -t_{\alpha, n-1}$	Probability below t_0	$d = (\mu_0 - \mu)/\sigma$	g, h

Problem 9.3.6

9.3.6 An article in the *ASCE Journal of Energy Engineering* (1999, Vol. 125, pp. 59–75) describes a study of the thermal inertia properties of autoclaved aerated concrete used as a building material. Five samples of the material were tested in a structure, and the average interior temperatures (°C) reported were as follows: 23.01, 22.22, 22.04, 22.62, and 22.59.

a. Test the hypotheses H_0: $\mu = 22.5$ versus H_1: $\mu \neq 22.5$, using $\alpha = 0.05$. Find the P-value.

b. Check the assumption that interior temperature is normally distributed.

c. Compute the power of the test if the true mean interior temperature is as high as 22.75.

d. What sample size would be required to detect a true mean interior temperature as high as 22.75 if you wanted the power of the test to be at least 0.9?

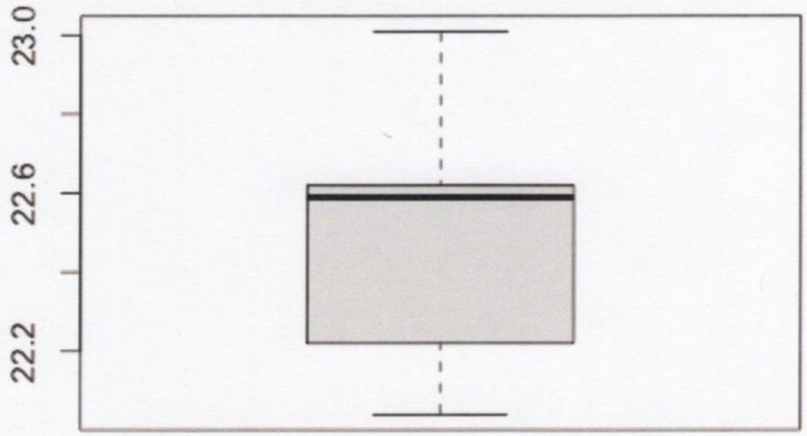
e. Explain how the question in part (a) could be answered by constructing a two-sided confidence interval on the mean interior temperature.

Descriptive Statistics

```
x<-c(23.01,22.22,22.04,22.62,22.59)
library(psych)
describe(x)

##    vars n mean   sd median trimmed  mad   min   max range skew kurtosis
se
## X1    1 5 22.5 0.38  22.59    22.5 0.55 22.04 23.01  0.97 0.08    -1.84
0.17
```

Boxplot

```
boxplot(x)
```



Classical Approach

Hypothesis Test Using R

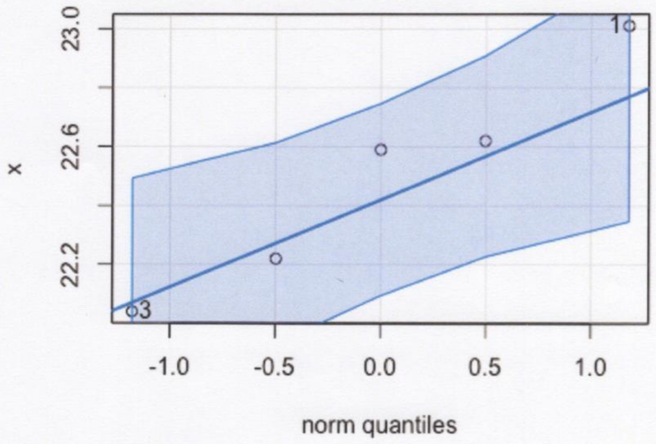
t-test

```
t.test(x,alternative="two.sided",mu=22.5,conf.level=0.95)

##
##  One Sample t-test
##
## data:  x
## t = -0.023642, df = 4, p-value = 0.9823
## alternative hypothesis: true mean is not equal to 22.5
## 95 percent confidence interval:
##  22.02625 22.96575
## sample estimates:
## mean of x
##    22.496
```

Normal Probability Plot

Normal Probability Plot

```
library(car)

## Loading required package: carData

##
## Attaching package: 'car'

## The following object is masked from 'package:psych':
##
##     logit

qqPlot(x)
```




```
## [1] 1 3
```

P-values

- More difficult to calculate since the t-tables only contain a few quantiles
- Can use tables to generate bounds on the p-value
- Software will provide p-values

P-values from R

t-test

```
t.test(x,alternative="two.sided",mu=22.5,conf.level=0.95)

##
##  One Sample t-test
##
## data:  x
## t = -0.023642, df = 4, p-value = 0.9823
## alternative hypothesis: true mean is not equal to 22.5
## 95 percent confidence interval:
##  22.02625 22.96575
## sample estimates:
## mean of x
##    22.496
```

Power Calculations

- Are much more complicated
- The true distribution is now a non-central t
- Use tables to solve (Chart VII in appendix) or software

Power Calculation using R

Power

```
power.t.test(n=5,delta=0.25,sd=0.38,sig.level=0.05,type="two.sample")

##
##      Two-sample t test power calculation
##
##              n = 5
##          delta = 0.25
##             sd = 0.38
##      sig.level = 0.05
##          power = 0.1491624
##    alternative = two.sided
##
## NOTE: n is number in *each* group
```

Sample Size using R

Sample Size

```
power.t.test(power=0.9,delta=0.25,sd=0.38,sig.level=0.05,type="two.sample")

##
##      Two-sample t test power calculation
##
##               n = 49.53305
##           delta = 0.25
##              sd = 0.38
##       sig.level = 0.05
##           power = 0.9
##     alternative = two.sided
##
## NOTE: n is number in *each* group
```

Chapter 9, Case 2 2-sided Practice Problems

Chapter 9, Case 2 Lower Practice Problems

Chapter 9, Case 2 Upper Practice Problems

Case 3. Hypothesis Test on Variance of Normal Population

The test statistics is a χ^2 random variable

$$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2}$$

Tests on the Variance of a Normal Distribution

Null hypothesis: H_0: $\sigma^2 = \sigma^2_0$

Test statistic: $\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2}$

Alternative Hypothesis	Rejection Criteria
$H_1 : \sigma^2 \neq \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha/2,n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$
$H_1 : \sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha,n-1}$
$H_1 : \sigma^2 < \sigma_0^2$	$\chi_0^2 < \chi^2_{1-\alpha,n-1}$

- The table below summarizes the three possible hypothesis tests. The rejection regions are clearly shown in Figure 9-17 on page 222

Figure 9-17

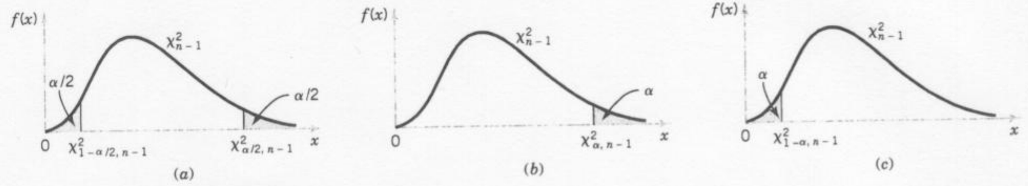


FIGURE 9.17

Reference distribution for the test of H_0: $\sigma^2 = \sigma_0^2$ with critical region values for (a) H_1: $\sigma^2 \neq \sigma_0^2$, (b) H_1: $\sigma^2 > \sigma_0^2$, and (c) H_1: $\sigma^2 < \sigma_0^2$.

Test Summary

See summary in your textbook

$H_0: \sigma^2 = \sigma_0^2$	$x_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$	$H_1: \sigma^2 \neq \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha/2,n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$	See text Section 9.4.	$\lambda = \sigma/\sigma_0$	*i, j*
		$H_1: \sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha,n-1}$		$\lambda = \sigma/\sigma_0$	*k, l*
		$H_1: \sigma^2 < \sigma_0^2$	$\chi_0^2 < \chi^2_{1-\alpha,n-1}$		$\lambda = \sigma/\sigma_0$	*m, n*

Problem 7.108

Problem taken from Ostle, Turner, Hicks and McElrath (1996). *Engineering Statistics: The Industrial Experience.* Duxbury Press.

7.108 Incoming coal at a coking plant is routinely analyzed for sulfur content (in percent). In the past, samples taken from barges loaded with coal from a particular mine have had a variance of 0.000196. When a new analyst was hired, the results of an assay of coal from the mine produced percentages of 0.83, 0.79, 0.77, 0.81, and 0.80.

(a) Using $\alpha = 0.05$, does the sample variance provide sufficient evidence to conclude that the results from the new analyst indicate more variability than in the past? State all assumptions.

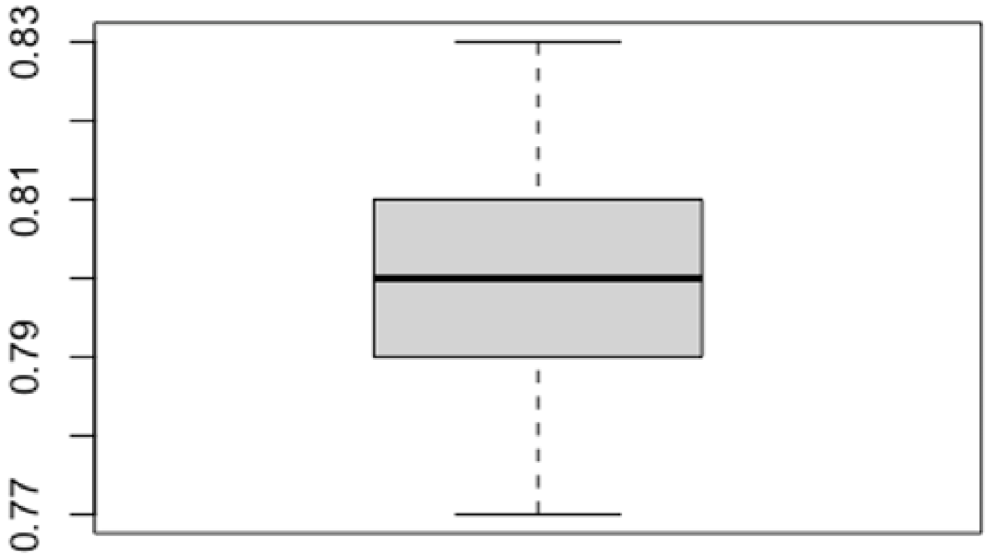
(b) Based on these data, is an assumption of normality reasonable? Justify by using a normal quantile plot *and* a formal test such as the Shapiro-Wilk W test.

Statistics for Problem 7.108

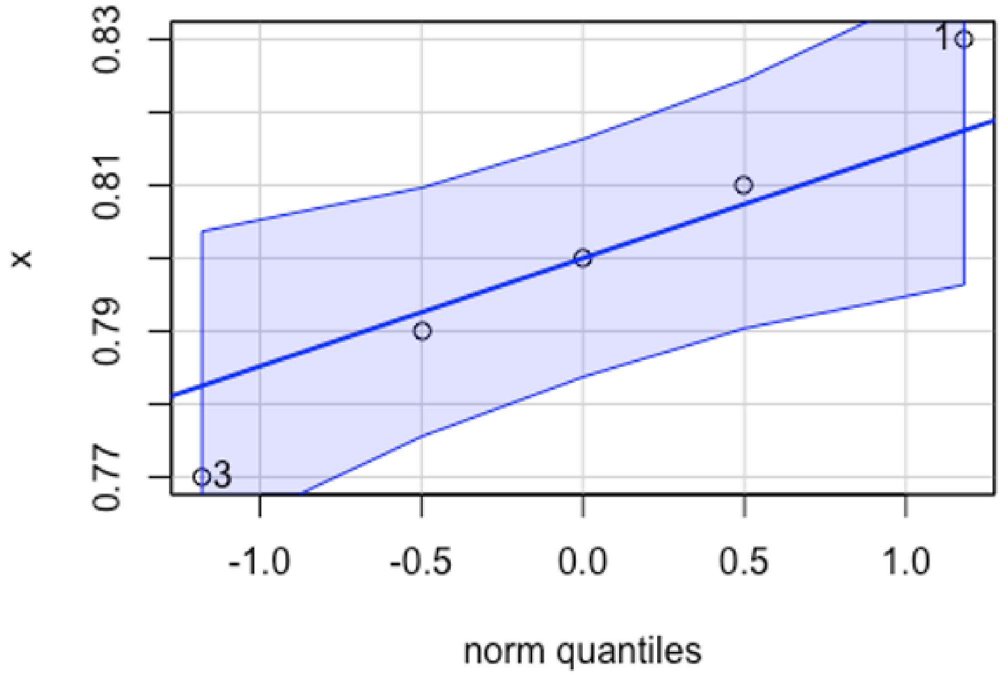
```
x<-c(0.83,0.79,0.77,0.81,0.80)
library(psych)
describe(x)

##    vars n mean   sd median trimmed  mad  min  max range skew kurtosis   se
## X1    1 5  0.8 0.02    0.8     0.8 0.01 0.77 0.83  0.06    0    -1.69 0.01

print(var(x))

## [1] 5e-04
```

Classical Approach

Problem 7.108 Plots



Problem 7.108 Plots



Problem 7.108 Shapiro-Wilks Test

```
## [1] 3 1

shapiro.test(x)

##
##  Shapiro-Wilk normality test
##
## data:  x
## W = 0.99929, p-value = 0.9998
```

***p*-values**

- Very similar to the case for the mean of a normal population with variance unknown
- Difficult to calculate since the χ^2-tables only contain a few quantiles
- Can use tables to generate bounds on the p-value
- Software will provide p-values

Test on Variance using R (EnvStats)

```
varTest(x,alternative="greater",conf.level=0.95,sigma.squared=0.000196)

##
## Results of Hypothesis Test
## --------------------------
##
## Null Hypothesis:                 variance = 0.000196
##
## Alternative Hypothesis:          True variance is greater than 0.000196
##
## Test Name:                       Chi-Squared Test on Variance
##
## Estimated Parameter(s):          variance = 5e-04
##
## Data:                            x
##
## Test Statistic:                  Chi-Squared = 10.20408
##
## Test Statistic Parameter:        df = 4
##
## P-value:                         0.03712675
##
## 95% Confidence Interval:         LCL = 0.0002107986
##                                  UCL =          Inf
```

Power Calculations

- Can be done with OC curves found in Table VII*i*–*n*
- Can be done in software such as R

Test on Standard Deviation

- What about test on standard deviation?

Chapter 9, Case 3 2-sided Practice Problems

Practice Problem Chapter 9, Case 3 Lower

Practice Problem Chapter 9, Case 3 Upper

Case 4. Hypothesis Test on a Population Proportion

- The test statistics for the hypothesis test is

$$Z_0 = \frac{x - np_0}{\sqrt{np_0(1 - p_0)}}$$

4.	$H_0: p = p_0$	$z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$	$H_1: p \neq p_0$	$\lvert z_0 \rvert > z_{\alpha/2}$	$p = 2[1 - \Phi(z_0)]$	3–4	3–4
			$H_1: p > p_0$	$z_0 > z_\alpha$	Probability above z_0 $p = 1 - \Phi(z_0)$	3–4	3–4
			$H_1: p < p_0$	$z_0 < -z_\alpha$	Probability below z_0 $P = \Phi(z_0)$	3–4	3–4

Problem 9.5.2

9.5.2 WP Suppose that of 1000 customers surveyed, 850 are satisfied or very satisfied with a corporation's products and services.

a. Test the hypothesis H_0: $p = 0.9$ against H_1: $p \neq 0.9$ at $\alpha = 0.05$. Find the P-value.

b. Explain how the question in part (a) could be answered by constructing a 95% two-sided confidence interval for p.

Problem 9.5.2 Classic Approach

Power Calculations

- For the two-sided alternative hypothesis

$$\beta = \Phi\left(\frac{p_0 - p + z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right) - \Phi\left(\frac{p_0 - p - z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right)$$

- If the alternative is $H_1: p < p_0$

$$\beta = 1 - \Phi\left(\frac{p_0 - p - z_{\alpha}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right)$$

- and finally if the alternative hypothesis is $H_1: p > p_0$

$$\beta = \Phi\left(\frac{p_0 - p + z_{\alpha}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right)$$

Sample Size

- Sample size requirements to satisfy type II(β) error constraints for a two-tailed hypothesis test is given by

$$n = \left[\frac{z_{\alpha/2}\sqrt{p_0(1-p_0)} + z_\beta\sqrt{p(1-p)}}{p - p_0}\right]^2.$$

- For a sample size for a one-sided test substitute z_α for $z_{\alpha/2}$.
- Problem 9.95

Testing for Goodness of Fit

- Material is presented in section 9-7 of your textbook
- Procedure determines if the sample data is from a specified underlying distribution
- Procedure uses a χ^2 distribution
- Example 9-12 presents a χ^2 goodness of fit test for a Poisson example
- Example 9-13 presents a χ^2 goodness of fit test for a normal example

Procedure

1. Collect a random sample of size n from a population with an unknown distribution,
2. Arrange the n observations in a frequency distribution containing k classes
3. Calculate the observed frequency in each class O_i,
4. From the hypothesized distribution, calculate the expected frequency in class i, denoted E_i (if E_i is small combine classes)
5. Calculate the test statistic

$$\chi_0^2 = \frac{\sum_{i=1}^{k}(O_i - E_i)^2}{E_i}$$

6. Reject the null hypothesis if the calculated value of the test statistic $\chi_0^2 > \chi_{\alpha,k-p-1}^2$ where p is the number of parameters in the hypothesized distribution

Example 9.12, part 1

EXAMPLE 9.12 | Printed Circuit Board Defects—Poisson Distribution

The number of defects in printed circuit boards is hypothesized to follow a Poisson distribution. A random sample of $n = 60$ printed circuit boards has been collected, and the following number of defects observed.

Number of Defects	Observed Frequency
0	32
1	15
2	9
3	4

The mean of the assumed Poisson distribution in this example is unknown and must be estimated from the sample data. The estimate of the mean number of defects per board is the sample average, that is, $(32 \cdot 0 + 15 \cdot 1 + 9 \cdot 2 + 4 \cdot 3)/60 = 0.75$. From the Poisson distribution with parameter 0.75, we may compute p_i, the theoretical, hypothesized probability associated with the ith class interval. Because each class interval corresponds to a particular number of defects, we may find the p_i as follows:

$$p_1 = P(X = 0) = \frac{e^{-0.75}(0.75)^0}{0!} = 0.472$$

$$p_2 = P(X = 1) = \frac{e^{-0.75}(0.75)^1}{1!} = 0.354$$

$$p_3 = P(X = 2) = \frac{e^{-0.75}(0.75)^2}{2!} = 0.133$$

$$p_4 = P(X \geq 3) = 1 - (p_1 + p_2 + p_3) = 0.041$$

Example 9.12, part 2

The expected frequencies are computed by multiplying the sample size $n = 60$ times the probabilities p_i. That is, $E_i = np_i$. The expected frequencies follow:

Number of Defects	Probability	Expected Frequency
0	0.472	28.32
1	0.354	21.24
2	0.133	7.98
3 (or more)	0.041	2.46

Because the expected frequency in the last cell is less than 3, we combine the last two cells:

Number of Defects	Observed Frequency	Expected Frequency
0	32	28.32
1	15	21.24
2 (or more)	13	10.44

The seven-step hypothesis-testing procedure may now be applied, using $\alpha = 0.05$, as follows:

1. **Parameter of interest:** The variable of interest is the form of the distribution of defects in printed circuit boards.
2. **Null hypothesis:** H_0: The form of the distribution of defects is Poisson.
3. **Alternative hypothesis:** H_1: The form of the distribution of defects is not Poisson.
4. **Test statistic:** The test statistic is $\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$
5. **Reject H_0 if:** Because the mean of the Poisson distribution was estimated, the preceding chi-square statistic will have $k - p - 1 = 3 - 1 - 1 = 1$ degree of freedom. Consider whether the P-value is less than 0.05.
6. **Computations:**

$$\chi_0^2 = \frac{(32 - 28.32)^2}{28.32} + \frac{(15 - 21.24)^2}{21.24} + \frac{(13 - 10.44)^2}{10.44} = 2.94$$

7. **Conclusions:** We find from Appendix Table III that $\chi^2_{0.10,1} = 2.71$ and $\chi^2_{0.05,1} = 3.84$. Because $\chi_0^2 = 2.94$ lies between these values, we conclude that the P-value is between 0.05 and 0.10. Therefore, because the P-value exceeds 0.05, we are unable to reject the null hypothesis that the distribution of defects in printed circuit boards is Poisson. The exact P-value computed from software is 0.0864.

Chapter 9 Summary

- You should be prepared to work any practice problems assigned: Cases 1-3 with three different alternatives
- All other information is conceptual knowledge that can be questioned with multiple choice
  – Name 3 ways to test if data is from a normal distribution

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\, du$$

TABLE III Cumulative Standard Normal Distribution

z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	−0.00
−3.9	0.000033	0.000034	0.000036	0.000037	0.000039	0.000041	0.000042	0.000044	0.000046	0.000048
−3.8	0.000050	0.000052	0.000054	0.000057	0.000059	0.000062	0.000064	0.000067	0.000069	0.000072
−3.7	0.000075	0.000078	0.000082	0.000085	0.000088	0.000092	0.000096	0.000100	0.000104	0.000108
−3.6	0.000112	0.000117	0.000121	0.000126	0.000131	0.000136	0.000142	0.000147	0.000153	0.000159
−3.5	0.000165	0.000172	0.000179	0.000185	0.000193	0.000200	0.000208	0.000216	0.000224	0.000233
−3.4	0.000242	0.000251	0.000260	0.000270	0.000280	0.000291	0.000302	0.000313	0.000325	0.000337
−3.3	0.000350	0.000362	0.000376	0.000390	0.000404	0.000419	0.000434	0.000450	0.000467	0.000483
−3.2	0.000501	0.000519	0.000538	0.000557	0.000577	0.000598	0.000619	0.000641	0.000664	0.000687
−3.1	0.000711	0.000736	0.000762	0.000789	0.000816	0.000845	0.000874	0.000904	0.000935	0.000968
−3.0	0.001001	0.001035	0.001070	0.001107	0.001144	0.001183	0.001223	0.001264	0.001306	0.001350
−2.9	0.001395	0.001441	0.001489	0.001538	0.001589	0.001641	0.001695	0.001750	0.001807	0.001866
−2.8	0.001926	0.001988	0.002052	0.002118	0.002186	0.002256	0.002327	0.002401	0.002477	0.002555
−2.7	0.002635	0.002718	0.002803	0.002890	0.002980	0.003072	0.003167	0.003264	0.003364	0.003467
−2.6	0.003573	0.003681	0.003793	0.003907	0.004025	0.004145	0.004269	0.004396	0.004527	0.004661
−2.5	0.004799	0.004940	0.005085	0.005234	0.005386	0.005543	0.005703	0.005868	0.006037	0.006210
−2.4	0.006387	0.006569	[illegible]	0.006947	0.007143	0.007344	0.007549	0.007760	0.007976	0.008198
−2.3	0.008424	0.008656	0.008894	0.009137	0.009387	0.009642	0.009903	0.010170	0.010444	0.010724
−2.2	0.011011	0.011304	0.011604	0.011911	0.012224	0.012545	0.012874	0.013209	0.013553	0.013903
−2.1	0.014262	0.014629	0.015003	0.015386	0.015778	0.016177	0.016586	0.017003	0.017429	0.017864
−2.0	0.018309	0.018763	0.019226	0.019699	0.020182	0.020675	0.021178	0.021692	0.022216	0.022750
−1.9	0.023295	0.023852	0.024419	0.024998	0.025588	0.026190	0.026803	0.027429	0.028067	0.028717
−1.8	0.029379	0.030054	0.030742	0.031443	0.032157	0.032884	0.033625	0.034379	0.035148	0.035930
−1.7	0.036727	0.037538	0.038364	0.039204	0.040059	0.040929	0.041815	0.042716	0.043633	0.044565
−1.6	0.045514	0.046479	0.047460	0.048457	0.049471	0.050503	0.051551	0.052616	0.053699	0.054799
−1.5	0.055917	0.057053	0.058208	0.059380	0.060571	0.061780	0.063008	0.064256	0.065522	0.066807
−1.4	0.068112	0.069437	0.070781	0.072145	0.073529	0.074934	0.076359	0.077804	0.079270	0.080757
−1.3	0.082264	0.083793	0.085343	0.086915	0.088508	0.090123	0.091759	0.093418	0.095098	0.096801
−1.2	0.098525	0.100273	0.102042	0.103835	0.105650	0.107488	0.109349	0.111233	0.113140	0.115070
−1.1	0.117023	0.119000	0.121001	0.123024	0.125072	0.127143	0.129238	0.131357	0.133500	0.135666
−1.0	0.137857	0.140071	0.142310	0.144572	0.146859	0.149170	0.151505	0.153864	0.156248	0.158655
−0.9	0.161087	0.163543	0.166023	0.168528	0.171056	0.173609	0.176185	0.178786	0.181411	0.184060
−0.8	0.186733	0.189430	0.192150	0.194894	0.197662	0.200454	0.203269	0.206108	0.208970	0.211855
−0.7	0.214764	0.217695	0.220650	0.223627	0.226627	0.229650	0.232695	0.235762	0.238852	0.241964
−0.6	0.245097	0.248252	0.251429	0.254627	0.257846	0.261086	0.264347	0.267629	0.270931	0.274253
−0.5	0.277595	0.280957	0.284339	0.287740	0.291160	0.294599	0.298056	0.301532	0.305026	0.308538
−0.4	0.312067	0.315614	0.319178	0.322758	0.326355	0.329969	0.333598	0.337243	0.340903	0.344578
−0.3	0.348268	0.351973	0.355691	0.359424	0.363169	0.366928	0.370700	0.374484	0.378281	0.382089
−0.2	0.385908	0.389739	0.393580	0.397432	0.401294	0.405165	0.409046	0.412936	0.416834	0.420740
−0.1	0.424655	0.428576	0.432505	0.436441	0.440382	0.444330	0.448283	0.452242	0.456205	0.460172
0.0	0.464144	0.468119	0.472097	0.476078	0.480061	0.484047	0.488033	0.492022	0.496011	0.500000

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\, du$$

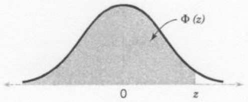


TABLE III **Cumulative Standard Normal Distribution (*continued*)**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

Table IV Percentage Points $t_{\alpha,\nu}$ of the t-Distribution

ν \ α	.40	.25	.10	.05	.025	.01	.005	.0025	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	.277	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	.255	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	.254	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	.254	.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	.253	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

ν = degrees of freedom.

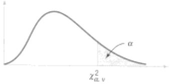


Table III Percentage Points $\chi^2_{\alpha,\nu}$ of the Chi-Squared Distribution

ν \ α	.995	.990	.975	.950	.900	.500	.100	.050	.025	.010	.005
1	.00+	.00+	.00+	.00+	.02	.45	2.71	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	.21	1.39	4.61	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	.58	2.37	6.25	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	2.20	5.35	10.65	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	10.34	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	13.34	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.27	7.26	8.55	14.34	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	15.34	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	16.34	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.87	17.34	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	24.34	34.28	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	25.34	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.65
28	12.46	13.57	15.31	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	59.33	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	69.33	85.53	90.53	95.02	100.42	104.22
80	51.17	53.54	57.15	60.39	64.28	79.33	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	89.33	107.57	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	99.33	118.50	124.34	129.56	135.81	140.17

ν = degrees of freedom.