Attendance : I-A

MANE 3332.05

Lecture 25

Agenda

- Continue Chapter 9 lecture
  - Chapter 9, Case 2
- Chapter 9, Case 1 2-sided Practice Problems (assigned 11/20/2025, due 11/25/2025)
- Chapter 9, Case 1 Lower Practice Problems (assigned 11/20/2025, due 11/25/2025)
- Chapter 9, Case 1 Upper Practice Problems (assigned 11/20/2025, due 11/25/2025)
- NEW: Chapter 9, Case 1 2-sided Quiz (assigned 11/25/2025, due 12/2/2025)
- NEW: Chapter 9, Case 1 Lower Quiz (assigned 11/25/2025, due 12/2/2025)
- NEW: Chapter 9, Case 1 Upper Quiz (assigned 11/25/2025, due 12/2/2025)
- NEW: Chapter 9, Case 2 2-sided Practice Problems (assigned 11/25/2025, due 12/2/2025)
- NEW: Chapter 9, Case 2 Lower Practice Problems (assigned 11/25/2025, due 12/2/2025)
- NEW: Chapter 9, Case 2 Upper Practice Problems (assigned 11/25/2025, due 12/2/2025)
- Attendance
- Questions?

Handouts

- Lecture 25 Slides
- Lecture 25 Slides - marked

Week	Tuesday Lecture	Thursday Lecture
13	**11/25** - Chapter 9, Case 2 (1)	**11/27** - Thanksgiving Holiday (no class)
14	**12/2** - Chapter 9, Case 3 (2)	**12/4** - Linear Regression (3)
15	**12/9** - Review Session (4)	**12/11** - Study Day (no class)

The final exam for MANE 3332.01 is **Thursday December 18, 2025 at 1:15 - 3:00 PM**.

Summary of One-Sample Hypothesis-Testing Procedures

Case	Null Hypothesis	Test Statistic	Alternative Hypothesis	Fixed Significance Level Criteria for Rejection	*P* - value	O.C. Curve Parameter	O.C. Curve Appendix Chart VII
1.	$H_0: \mu = \mu_0$ σ^2 known	$z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$H_1: \mu \neq \mu_0$	$\lvert z_0 \rvert > z_{\alpha/2}$	$P = 2[1 - \Phi(\lvert z_0 \rvert)]$	$d = \lvert \mu - \mu_0 \rvert / \sigma$	a, b
			$H_1: \mu > \mu_0$	$z_0 > z_\alpha$	Probability above z_0 $P = 1 - \Phi(z_0)$	$d = (\mu - \mu_0)/\sigma$	c, d
			$H_1: \mu < \mu_0$	$z_0 < -z_\alpha$	Probability below z_0 $P = \Phi(z_0)$	$d = (\mu_0 - \mu)/\sigma$	c, d
2.	$H_0: \mu = \mu_0$ σ^2 unknown	$t_0 = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$H_1: \mu \neq \mu_0$	$\lvert t_0 \rvert > t_{\alpha/2, n-1}$	Sum of the probability above $\lvert t_0 \rvert$ and below $-\lvert t_0 \rvert$	$d = \lvert \mu - \mu_0 \rvert / \sigma$	e, f
			$H_1: \mu > \mu_0$	$t_0 > t_{\alpha, n-1}$	Probability above t_0	$d = (\mu - \mu_0)/\sigma$	g, h
			$H_1: \mu < \mu_0$	$t_0 < -t_{\alpha, n-1}$	Probability below t_0	$d = (\mu_0 - \mu)/\sigma$	g, h
3.	$H_0: \sigma^2 = \sigma_0^2$	$x_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$	$H_1: \sigma^2 \neq \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha/2, n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2, n-1}$	See text Section 9.4.	$\lambda = \sigma/\sigma_0$	i, j
			$H_1: \sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha, n-1}$		$\lambda = \sigma/\sigma_0$	k, l
			$H_1: \sigma^2 < \sigma_0^2$	$\chi_0^2 < \chi^2_{1-\alpha, n-1}$		$\lambda = \sigma/\sigma_0$	m, n
4.	$H_0: p = p_0$	$z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$	$H_1: p \neq p_0$	$\lvert z_0 \rvert > z_{\alpha/2}$	$p = 2[1 - \Phi(\lvert z_0 \rvert)]$	3–4	3–4
			$H_1: p > p_0$	$z_0 > z_\alpha$	Probability above z_0 $p = 1 - \Phi(z_0)$	3–4	3–4
			$H_1: p < p_0$	$z_0 < -z_\alpha$	Probability below z_0 $P = \Phi(z_0)$	3–4	3–4

Summary of One-Sample Confidence Interval Procedures

Case	Problem Type	Point Estimate	Two-sided $100(1-\alpha)$ Percent Confidence Interval
1.	Mean μ, variance σ^2 known	$\bar{x}$	$\bar{x} - z_{\alpha/2}\sigma/\sqrt{n} \le \mu \le \bar{x} + z_{\alpha/2}\sigma/\sqrt{n}$
2.	Mean μ of a normal distribution, variance σ^2 unknown	$\bar{x}$	$\bar{x} - t_{\alpha/2, n-1}s/\sqrt{n} \le \mu \le \bar{x} + t_{\alpha/2, n-1}s/\sqrt{n}$
3.	Variance σ^2 of a normal distribution	s^2	$\dfrac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}} \le \sigma^2 \le \dfrac{(n-1)s^2}{\chi^2_{1-\alpha/2, n-1}}$
4.	Proportion or parameter of a binomial distribution p	$\hat{p}$	$\hat{p} - z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

Chapter 9, Case 2

Hypothesis Test on the Mean, Variance Unknown

- Much more common case than variance known
- Substitute S for σ
- The test statistics is now a t random variable

practical

$$T = \frac{\overline{X} - \mu_0}{S/\sqrt{n}}$$

Summary of Case 2

$H_0: \mu = \mu_0$ σ^2 unknown	$t_0 = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$H_1: \mu \neq \mu_0$	$\lvert t_0 \rvert > t_{\alpha/2, n-1}$	Sum of the probability above $\lvert t_0 \rvert$ and below $-\lvert t_0 \rvert$	$d = \lvert \mu - \mu_0 \rvert / \sigma$	e, f
		$H_1: \mu > \mu_0$	$t_0 > t_{\alpha, n-1}$	Probability above t_0	$d = (\mu - \mu_0)/\sigma$	g, h
		$H_1: \mu < \mu_0$	$t_0 < -t_{\alpha, n-1}$	Probability below t_0	$d = (\mu_0 - \mu)/\sigma$	g, h

$H_0: \mu = 22.5$

$H_1: \mu \neq 22.5$

Problem 9.3.6

Test Statistics

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$= \frac{22.5 - 22.5}{.38/\sqrt{n}}$$

$$= 0.0$$

9.3.6 An article in the *ASCE Journal of Energy Engineering* (1999, Vol. 125, pp. 59–75) describes a study of the thermal inertia properties of autoclaved aerated concrete used as a building material. Five samples of the material were tested in a structure, and the average interior temperatures (°C) reported were as follows: 23.01, 22.22, 22.04, 22.62, and 22.59.

a. Test the hypotheses H_0: $\mu = 22.5$ versus H_1: $\mu \neq 22.5$, using $\alpha = 0.05$. Find the P-value.

b. Check the assumption that interior temperature is normally distributed.

c. Compute the power of the test if the true mean interior temperature is as high as 22.75.

d. What sample size would be required to detect a true mean interior temperature as high as 22.75 if you wanted the power of the test to be at least 0.9?

e. Explain how the question in part (a) could be answered by constructing a two-sided confidence interval on the mean interior temperature.

Descriptive Statistics

```
x<-c(23.01,22.22,22.04,22.62,22.59)
library(psych)
describe(x)

##     vars n mean   sd median trimmed  mad   min   max range skew kurtosis
se
## X1     1 5 22.5 0.38  22.59    22.5 0.55 22.04 23.01  0.97 0.08    -1.84
0.17
```

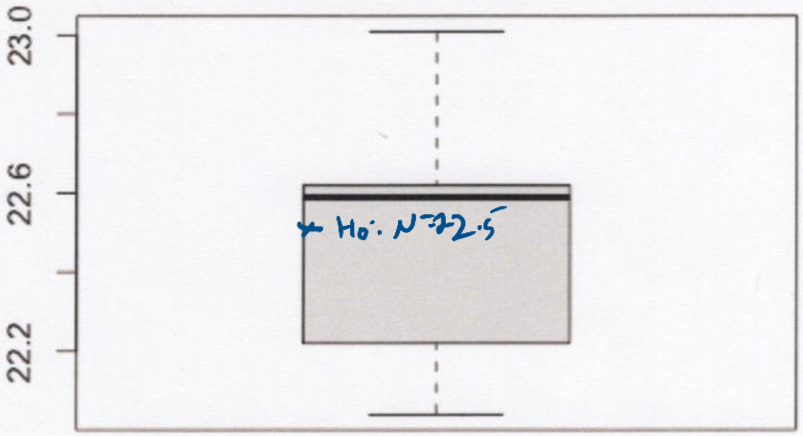
Rejection Region

Reject H_0 if $|t_0| > t_{\alpha/2, n-1} = t_{.025,4} = 2.776$

Conclusion: Since $|t_0|$ is not greater than 2.776, we fail to reject H_0

Boxplot

```
boxplot(x)
```



Classical Approach

Hypothesis Test

t-test

```
t.test(x,alternative="two.sided",mu=22.5,conf.level=0.95)
```

```
##
##  One Sample t-test
##
## data:  x
## t = -0.023642, df = 4, p-value = 0.9823
## alternative hypothesis: true mean is not equal to 22.5
## 95 percent confidence interval:
##  22.02625 22.96575
## sample estimates:
## mean of x
##    22.496
```

if H_0 is in the confidence interval, fail to reject H_0

Normal Probability Plot

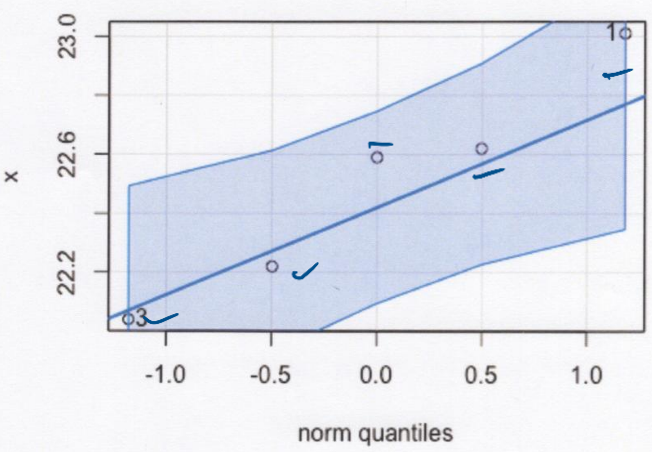
H_0: data is normal

H_1: data is not normal

If all points are within the bounds, we fail to reject H_0

Normal Probability Plot

```
library(car)

## Loading required package: carData

##
## Attaching package: 'car'

## The following object is masked from 'package:psych':
##
##     logit

qqPlot(x)
```




```
## [1] 1 3
```

P-values

- More difficult to calculate since the t-tables only contain a few quantiles
- Can use tables to generate bounds on the p-value
- Software will provide p-values

P-values from R

t-test

```
t.test(x,alternative="two.sided",mu=22.5,conf.level=0.95)

##
##  One Sample t-test
##
## data:  x
## t = -0.023642, df = 4, p-value = 0.9823
## alternative hypothesis: true mean is not equal to 22.5
## 95 percent confidence interval:
##  22.02625 22.96575
## sample estimates:
## mean of x
##    22.496
```

Power Calculations

- Are much more complicated
- The true distribution is now a non-central t
- Use tables to solve (Chart VII in appendix) or software

Power Calculation using R

Power

```
power.t.test(n=5,delta=0.25,sd=0.38,sig.level=0.05,type="two.sample")

##
##      Two-sample t test power calculation
##
##               n = 5
##           delta = 0.25
##              sd = 0.38
##       sig.level = 0.05
##           power = 0.1491624
##     alternative = two.sided
##
## NOTE: n is number in *each* group
```

Sample Size using R

Sample Size

```
power.t.test(power=0.9,delta=0.25,sd=0.38,sig.level=0.05,type="two.sample")

##
##      Two-sample t test power calculation
##
##               n = 49.53305
##           delta = 0.25
##              sd = 0.38
##       sig.level = 0.05
##           power = 0.9
##     alternative = two.sided
##
## NOTE: n is number in *each* group
```

Chapter 9, Case 2 2-sided Practice Problems

Question 1 (2 points)

Listen

Calculate the test statistic for a test of hypothesis for the mean of single sample with variance unknown. The null hypothesis is mu=32.0 versus mu not equal to 32.0 using alpha=0.002. The sample statistics are n=19, xbar=33.78, S=4.808.

- t0=-1.6137
- t0=7.0341
- t0=1.463
- t0=1.6137

t_0, μ_0

$$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{33.78 - 32}{4.808/\sqrt{19}}$$

$$= 1.61374$$

Question 2 (2 points)

Listen

Construct the rejection region for a test of hypothesis for the mean of single sample with variance unknown. The null hypothesis is mu=25.8 versus mu not equal to 25.8 using alpha=0.2. The sample statistics are n=22, xbar=28.65, S=7.411.

- ○ Reject Ho if t0<-0.859
- ○ Reject H0 if |t0|>0.859
- ○ Reject Ho if t0<-1.323
- ○ Reject Ho if t0>1.323
- ✓ Reject H0 if |t0|>1.323
- ○ Reject Ho if t0>0.859

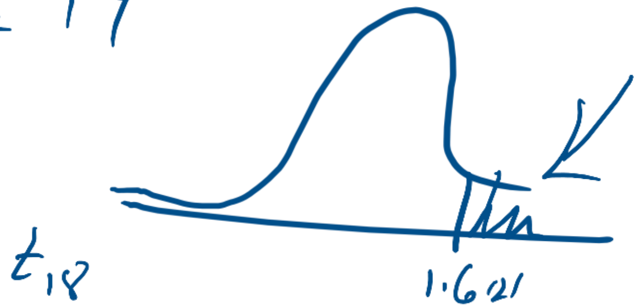
Reject Ho if

$|t_0| > t_{\alpha/2, n-1} = t_{.1, 21}$

$= 1.323$

P-value Question, Question 1

$t_0 = 1.614, \; n = 19$



Question 3 (2 points)

Listen

Which is the correct conclusion for a test of hypothesis for the mean of single sample with variance unknown. The null hypothesis is mu=132.3 versus mu not equal to 132.3 using alpha=0.001. The sample statistics are n=2, xbar=144.97, S=0.301. The value of t0 is 59.5285 and the rejection region is reject HO if |t0|>636.619

- ( ) Fail to reject H0
- ( ) Reject H0

Reject Ho if $|t_0| > 636.619$

Is $59.5285 > 636.619$?

no, fail to reject Ho

Question 4 (2 points)

Listen

Using the p-value from a test of hypothesis for the mean of single sample with variance unknown, determine the correct conclusion for the hypothesis test. The null hypothesis is mu=132.3 versus mu not equal to 132.3 using alpha=0.05. The sample statistics are n=5, xbar=130.75, S=15.112. The results of hypothesis test include t0= -0.2293 and p-value=0.829883.

- [x] Fail to reject H0
- [ ] Reject H0

p-value = .829883

α = .05

is p-value < α ? no, fail to Reject H0

Chapter 9, Case 2 Lower Practice Problems

Question 2 (2 points)

Listen

Construct the rejection region for a test of hypothesis for the mean of single sample with variance unknown. The null hypothesis is mu=0.5 versus mu less than 0.5 using alpha=0.01. The sample statistics are n=12, xbar=0.48, S=0.062.

- Reject Ho if t0<-3.106
- Reject H0 if |t0|>2.718
- Reject H0 if t0<-2.718
- Reject Ho if t0>2.718
- Reject Ho if |t0|>3.106

$H_1: \mu < 0.5$

Reject Ho if $t_0 < -t_{\alpha, n-1} = t_{.01, 11} = 2.718$

Chapter 9, Case 2 Upper Practice Problems

Question 2 (2 points)

Listen

Construct the rejection region for a test of hypothesis for the mean of single sample with variance unknown. The null hypothesis is mu=32.0 versus mu greater than 32.0 using alpha=0.005. The sample statistics are n=16, xbar=33.19, S=3.313.

- ◯ Reject Ho if |t0|>3.286
- ◯ Reject Ho if t0<-2.947
- ◯ Reject H0 if |t0|>2.947
- ◯ Reject Ho if t0<-3.286
- ✓ Reject H0 if t0>2.947

$H_1: \mu > 32$

Reject Ho if

$t_0 > t_{\alpha, n-1} = t_{.005, 15}$

$= 2.947$

Case 3. Hypothesis Test on Variance of Normal Population

The test statistics is a χ^2 random variable

$$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2}$$

Tests on the Variance of a Normal Distribution

Null hypothesis: $H_0\colon \sigma^2 = \sigma_0^2$

Test statistic: $\chi_0^2 = \dfrac{(n-1)S^2}{\sigma_0^2}$

Alternative Hypothesis	Rejection Criteria
$H_1 : \sigma^2 \neq \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha/2,n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$
$H_1 : \sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha,n-1}$
$H_1 : \sigma^2 < \sigma_0^2$	$\chi_0^2 < \chi^2_{1-\alpha,n-1}$

- The table below summarizes the three possible hypothesis tests. The rejection regions are clearly shown in Figure 9-17 on page 222

Figure 9-17

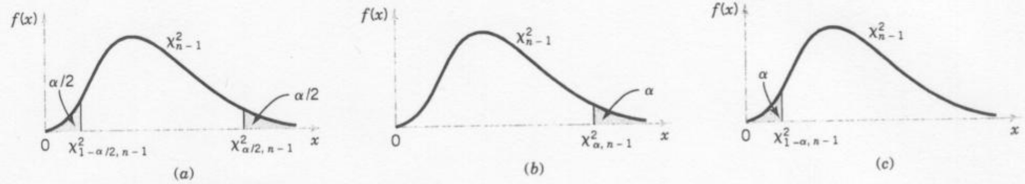


FIGURE 9.17

Reference distribution for the test of H_0: $\sigma^2 = \sigma_0^2$ with critical region values for (a) H_1: $\sigma^2 \neq \sigma_0^2$, (b) H_1: $\sigma^2 > \sigma_0^2$, and (c) H_1: $\sigma^2 < \sigma_0^2$.

Test Summary

See summary in your textbook

$H_0: \sigma^2 = \sigma_0^2$	$x_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$	$H_1: \sigma^2 \neq \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha/2,n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$	See text Section 9.4.	$\lambda = \sigma/\sigma_0$	*i, j*
		$H_1: \sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha,n-1}$		$\lambda = \sigma/\sigma_0$	*k, l*
		$H_1: \sigma^2 < \sigma_0^2$	$\chi_0^2 < \chi^2_{1-\alpha,n-1}$		$\lambda = \sigma/\sigma_0$	*m, n*

Problem 7.108

Problem taken from Ostle, Turner, Hicks and McElrath (1996). *Engineering Statistics: The Industrial Experience.* Duxbury Press.

7.108 Incoming coal at a coking plant is routinely analyzed for sulfur content (in percent). In the past, samples taken from barges loaded with coal from a particular mine have had a variance of 0.000196. When a new analyst was hired, the results of an assay of coal from the mine produced percentages of 0.83, 0.79, 0.77, 0.81, and 0.80.

(a) Using $\alpha = 0.05$, does the sample variance provide sufficient evidence to conclude that the results from the new analyst indicate more variability than in the past? State all assumptions.

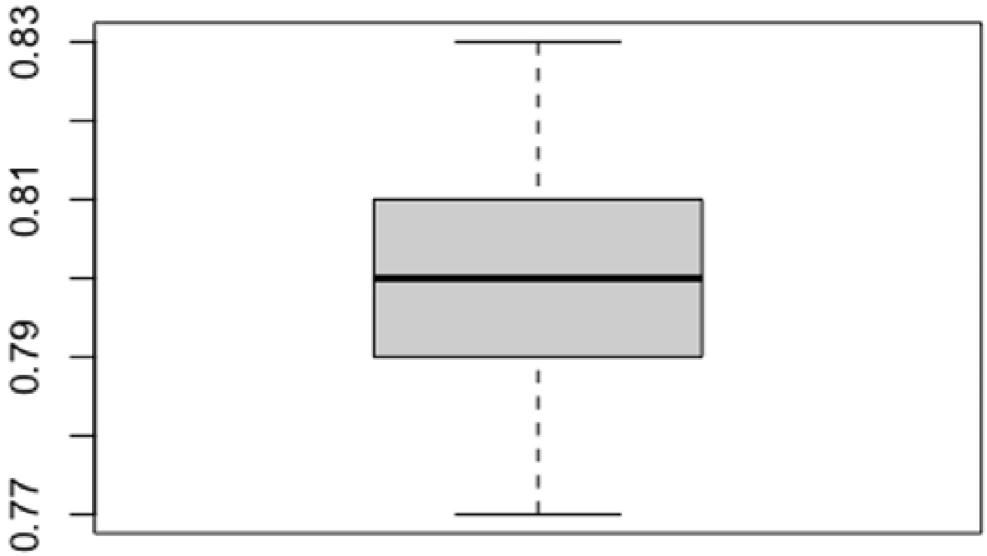
(b) Based on these data, is an assumption of normality reasonable? Justify by using a normal quantile plot *and* a formal test such as the Shapiro-Wilk W test.

Statistics for Problem 7.108

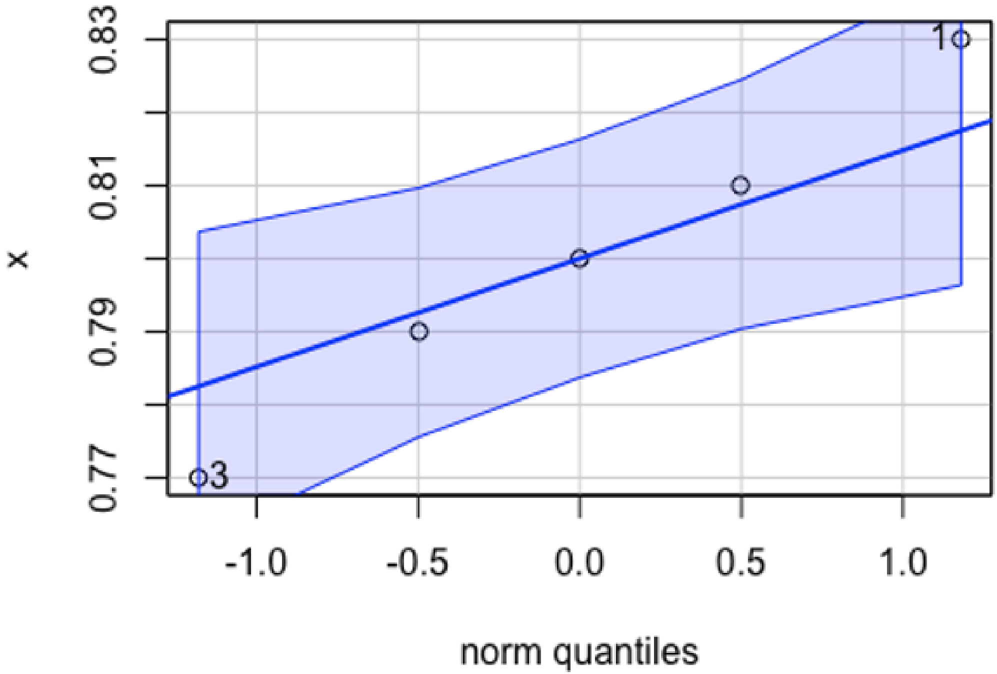
```
x<-c(0.83,0.79,0.77,0.81,0.80)
library(psych)
describe(x)

##    vars n mean   sd median trimmed  mad  min  max range skew kurtosis   se
## X1    1 5  0.8 0.02    0.8     0.8 0.01 0.77 0.83  0.06    0    -1.69 0.01

print(var(x))

## [1] 5e-04
```

Classical Approach

Problem 7.108 Plots



Problem 7.108 Plots



Problem 7.108 Shapiro-Wilks Test

```
## [1] 3 1

shapiro.test(x)

##
##  Shapiro-Wilk normality test
##
## data:  x
## W = 0.99929, p-value = 0.9998
```

***p*-values**

- Very similar to the case for the mean of a normal population with variance unknown
- Difficult to calculate since the χ^2-tables only contain a few quantiles
- Can use tables to generate bounds on the p-value
- Software will provide p-values

Test on Variance using R (EnvStats)

```
varTest(x,alternative="greater",conf.level=0.95,sigma.squared=0.000196)

##
## Results of Hypothesis Test
## --------------------------
##
## Null Hypothesis:                 variance = 0.000196
##
## Alternative Hypothesis:          True variance is greater than 0.000196
##
## Test Name:                       Chi-Squared Test on Variance
##
## Estimated Parameter(s):          variance = 5e-04
##
## Data:                            x
##
## Test Statistic:                  Chi-Squared = 10.20408
##
## Test Statistic Parameter:        df = 4
##
## P-value:                         0.03712675
##
## 95% Confidence Interval:         LCL = 0.0002107986
##                                  UCL =          Inf
```

Power Calculations

- Can be done with OC curves found in Table VII*i*–*n*
- Can be done in software such as R

Test on Standard Deviation

- What about test on standard deviation?

Chapter 9, Case 3 2-sided Practice Problems

Practice Problem Chapter 9, Case 3 Lower

Practice Problem Chapter 9, Case 3 Upper

Case 4. Hypothesis Test on a Population Proportion

- The test statistics for the hypothesis test is

$$Z_0 = \frac{x - np_0}{\sqrt{np_0(1 - p_0)}}$$

4.	$H_0: p = p_0$	$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}}$	$H_1: p \neq p_0$	$\lvert z_0 \rvert > z_{\alpha/2}$	$p = 2[1 - \Phi(z_0)]$	3–4	3–4
			$H_1: p > p_0$	$z_0 > z_\alpha$	Probability above z_0 $p = 1 - \Phi(z_0)$	3–4	3–4
			$H_1: p < p_0$	$z_0 < -z_\alpha$	Probability below z_0 $P = \Phi(z_0)$	3–4	3–4

Problem 9.5.2

9.5.2 WP Suppose that of 1000 customers surveyed, 850 are satisfied or very satisfied with a corporation's products and services.

a. Test the hypothesis H_0: $p = 0.9$ against H_1: $p \neq 0.9$ at $\alpha = 0.05$. Find the P-value.

b. Explain how the question in part (a) could be answered by constructing a 95% two-sided confidence interval for p.

Problem 9.5.2 Classic Approach

Power Calculations

- For the two-sided alternative hypothesis

$$\beta = \Phi\left(\frac{p_0 - p + z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right) - \Phi\left(\frac{p_0 - p - z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right)$$

- If the alternative is $H_1: p < p_0$

$$\beta = 1 - \Phi\left(\frac{p_0 - p - z_{\alpha}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right)$$

- and finally if the alternative hypothesis is $H_1: p > p_0$

$$\beta = \Phi\left(\frac{p_0 - p + z_{\alpha}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right)$$

Sample Size

- Sample size requirements to satisfy type II(β) error constraints for a two-tailed hypothesis test is given by

$$n = \left[\frac{z_{\alpha/2}\sqrt{p_0(1-p_0)} + z_\beta\sqrt{p(1-p)}}{p - p_0}\right]^2.$$

- For a sample size for a one-sided test substitute z_α for $z_{\alpha/2}$.
- Problem 9.95

Testing for Goodness of Fit

- Material is presented in section 9-7 of your textbook
- Procedure determines if the sample data is from a specified underlying distribution
- Procedure uses a χ^2 distribution
- Example 9-12 presents a χ^2 goodness of fit test for a Poisson example
- Example 9-13 presents a χ^2 goodness of fit test for a normal example

Procedure

1. Collect a random sample of size n from a population with an unknown distribution,
2. Arrange the n observations in a frequency distribution containing k classes
3. Calculate the observed frequency in each class O_i,
4. From the hypothesized distribution, calculate the expected frequency in class i, denoted E_i (if E_i is small combine classes)
5. Calculate the test statistic

$$\chi_0^2 = \frac{\sum_{i=1}^{k}(O_i - E_i)^2}{E_i}$$

6. Reject the null hypothesis if the calculated value of the test statistic $\chi_0^2 > \chi_{\alpha,k-p-1}^2$ where p is the number of parameters in the hypothesized distribution

Example 9.12, part 1

EXAMPLE 9.12 | Printed Circuit Board Defects—Poisson Distribution

The number of defects in printed circuit boards is hypothesized to follow a Poisson distribution. A random sample of $n = 60$ printed circuit boards has been collected, and the following number of defects observed.

Number of Defects	Observed Frequency
0	32
1	15
2	9
3	4

The mean of the assumed Poisson distribution in this example is unknown and must be estimated from the sample data. The estimate of the mean number of defects per board is the sample average, that is, $(32 \cdot 0 + 15 \cdot 1 + 9 \cdot 2 + 4 \cdot 3)/60 = 0.75$. From the Poisson distribution with parameter 0.75, we may compute p_i, the theoretical, hypothesized probability associated with the ith class interval. Because each class interval corresponds to a particular number of defects, we may find the p_i as follows:

$$p_1 = P(X = 0) = \frac{e^{-0.75}(0.75)^0}{0!} = 0.472$$

$$p_2 = P(X = 1) = \frac{e^{-0.75}(0.75)^1}{1!} = 0.354$$

$$p_3 = P(X = 2) = \frac{e^{-0.75}(0.75)^2}{2!} = 0.133$$

$$p_4 = P(X \geq 3) = 1 - (p_1 + p_2 + p_3) = 0.041$$

Example 9.12, part 2

The expected frequencies are computed by multiplying the sample size $n = 60$ times the probabilities p_i. That is, $E_i = np_i$. The expected frequencies follow:

Number of Defects	Probability	Expected Frequency
0	0.472	28.32
1	0.354	21.24
2	0.133	7.98
3 (or more)	0.041	2.46

Because the expected frequency in the last cell is less than 3, we combine the last two cells:

Number of Defects	Observed Frequency	Expected Frequency
0	32	28.32
1	15	21.24
2 (or more)	13	10.44

The seven-step hypothesis-testing procedure may now be applied, using $\alpha = 0.05$, as follows:

1. **Parameter of interest:** The variable of interest is the form of the distribution of defects in printed circuit boards.
2. **Null hypothesis:** H_0: The form of the distribution of defects is Poisson.
3. **Alternative hypothesis:** H_1: The form of the distribution of defects is not Poisson.
4. **Test statistic:** The test statistic is $\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$
5. **Reject H_0 if:** Because the mean of the Poisson distribution was estimated, the preceding chi-square statistic will have $k - p - 1 = 3 - 1 - 1 = 1$ degree of freedom. Consider whether the P-value is less than 0.05.
6. **Computations:**

$$\chi_0^2 = \frac{(32 - 28.32)^2}{28.32} + \frac{(15 - 21.24)^2}{21.24} + \frac{(13 - 10.44)^2}{10.44} = 2.94$$

7. **Conclusions:** We find from Appendix Table III that $\chi^2_{0.10,1} = 2.71$ and $\chi^2_{0.05,1} = 3.84$. Because $\chi_0^2 = 2.94$ lies between these values, we conclude that the P-value is between 0.05 and 0.10. Therefore, because the P-value exceeds 0.05, we are unable to reject the null hypothesis that the distribution of defects in printed circuit boards is Poisson. The exact P-value computed from software is 0.0864.

Chapter 9 Summary

- You should be prepared to work any practice problems assigned: Cases 1-3 with three different alternatives
- All other information is conceptual knowledge that can be questioned with multiple choice
  - Name 3 ways to test if data is from a normal distribution

Table IV Percentage Points $t_{\alpha,\nu}$ of the t-Distribution

ν \ α	.40	.25	.10	.05	.025	.01	.005	.0025	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	.277	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	.255	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	.254	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	.254	.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	.253	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

ν = degrees of freedom.

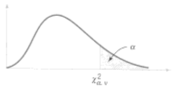


Table III Percentage Points $\chi^2_{\alpha,\nu}$ of the Chi-Squared Distribution

ν \ α	.995	.990	.975	.950	.900	.500	.100	.050	.025	.010	.005
1	.00+	.00+	.00+	.00+	.02	.45	2.71	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	.21	1.39	4.61	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	.58	2.37	6.25	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	2.20	5.35	10.65	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	10.34	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	13.34	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.27	7.26	8.55	14.34	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	15.34	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	16.34	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.87	17.34	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	24.34	34.28	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	25.34	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.65
28	12.46	13.57	15.31	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	59.33	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	69.33	85.53	90.53	95.02	100.42	104.22
80	51.17	53.54	57.15	60.39	64.28	79.33	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	89.33	107.57	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	99.33	118.50	124.34	129.56	135.81	140.17

ν = degrees of freedom.