MANE 3332.05

LECTURE 5

Agenda

- Continue Chapter 2 Lecture
- Start with Two Events Practice Problems
- Single Event Quiz (assigned 9/11/2025, due 9/15/2025)
- Two Events Practice Problems (assigned 9/16/2025, due 9/18/2025)

Handouts

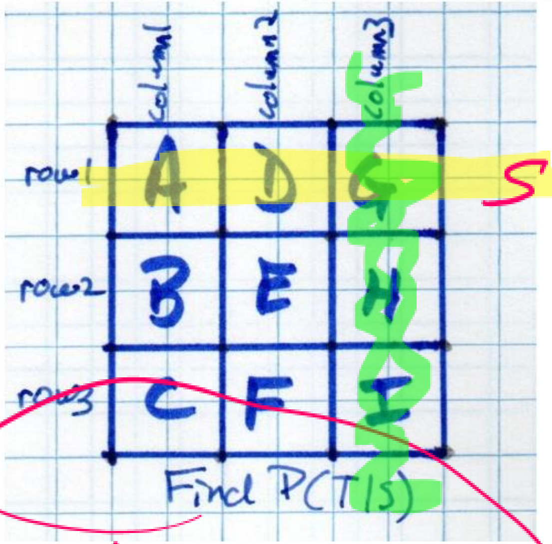
- Lecture 5 Slides - Powerpoint
- Lecture 5 Slides - Marked (pdf)

Two Events Practice Problems

Question 1 (1 point)

Listen

Consider a problem classified by 3 rows and 3 columns containing 500 observations. The table is described in the figure below and has the following cell counts: A=156, B=70, C=1, D=63, E=48, F=0, G=153, H=8, and I=1. Let event S denote an item that occurs in row 1 and event T denote an item that occurs in column 3. Find P(T|S).



S

T

$n = 500$

$$P(T|S) = \frac{P(T \cap S)}{P(S)}$$

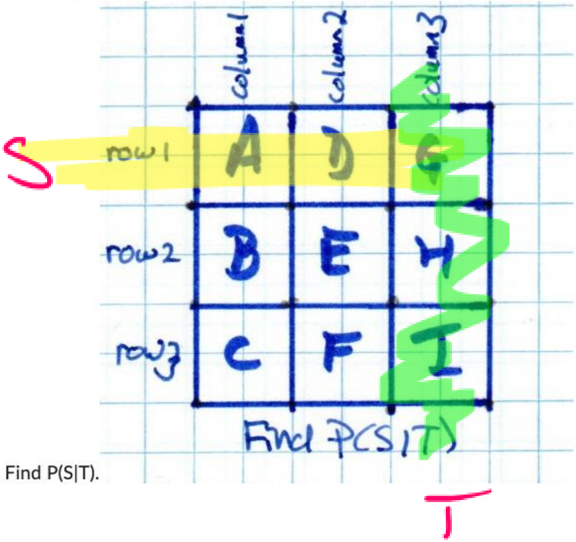
$$P(G \cap S) = \frac{G}{n} = \frac{153}{500} = .3060$$

$$P(S) = \frac{A+D+G}{n} = \frac{156+63+153}{500} = .744$$

$$P(T|S) = \frac{.3060}{.744} = .41129$$

S	156	63	153
	70	48	8
	1	0	1
			T

Consider a problem classified by 3 rows and 3 columns containing 2000 observations. The table is described in the figure below and has the following cell counts: A=756, B=368, C=6, D=130, E=48, F=1, G=551, H=137, and I=3. Let event S denote an item that occurs in row 1 and event T denote an item that occurs in



column 3. Find P(S|T).

$n = 2000$

$$P(S|T) = \frac{P(S \cap T)}{P(T)}$$

$$P(S \cap T) = \frac{G}{n} = \frac{551}{2000} = .2755$$

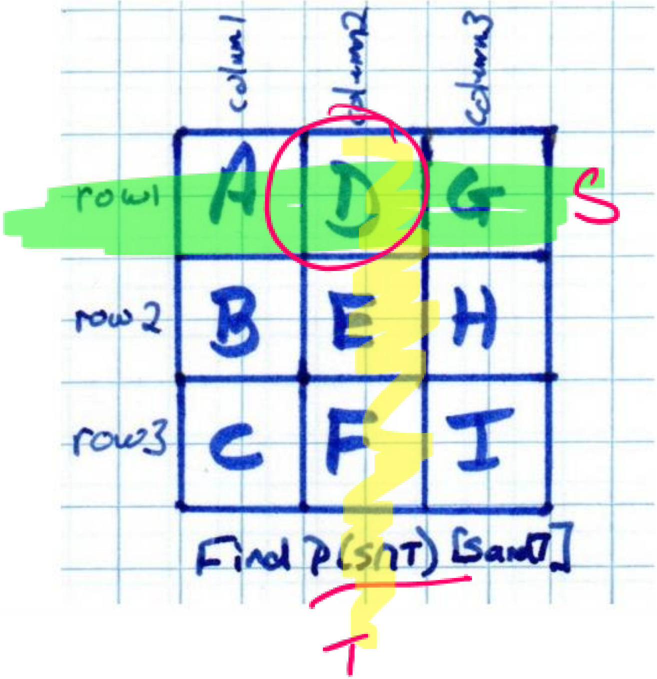
$$P(T) = \frac{G+H+I}{n} = \frac{551+137+3}{2000} = .3455$$

$$P(S|T) = \frac{.2755}{.3455} = .7974$$

Question 5 (1 point)

Listen

Consider a problem classified by 3 rows and 3 columns containing 300 observations. The table is described in the figure below and has the following cell counts: A=124, B=16, C=5, D=85, E=9, F=1, G=50, H=9, I=1. Let event S denote an item that occurs in row 1 and event T denote an item that occurs in column 2. Find P(S and T).



$n = 300$

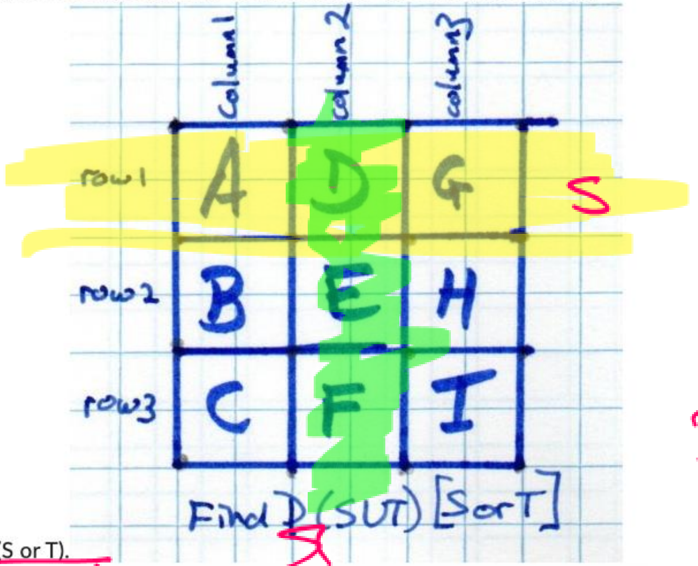
$$P(S \cap T) = \frac{D}{n} = \frac{\overset{85}{\cancel{50}}}{300}$$

$$= .2833$$

Question 7 (1 point)

Listen

Consider a problem classified by 3 rows and 3 columns containing 2000 observations. The table is described in the figure below and has the following cell counts: A=1666, B=1, C=0, D=302, E=1, F=1, G=29, H=0, and I=0. Let event S denote an item that occurs in row 1 and event T denote an item that occurs in



column 2. Find P(S or T).

$n = 2000$

$P(S \cup T) = P(S) + P(T) - P(S \cap T)$

$P(S) = \frac{A+D+G}{n} = \frac{1666+302+29}{2000} = \frac{1997}{2000} = .99850$

$P(T) = \frac{D+E+F}{n} = \frac{302+1+1}{2000} = .152$

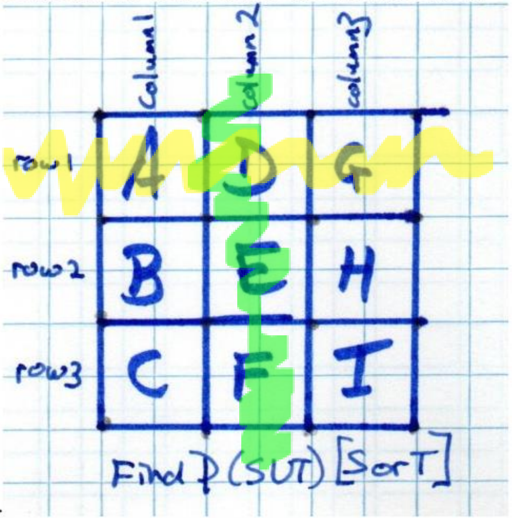
$P(S \cap T) = \frac{D}{n} = \frac{302}{2000} = .151$

$P(S \cup T) = .99850 + .152 - .151 = .99950$

Question 7 (1 point)

Listen

Consider a problem classified by 3 rows and 3 columns containing 2000 observations. The table is described in the figure below and has the following cell counts: A=1666, B=1, C=0, D=302, E=1, F=1, G=29, H=0, and I=0. Let event S denote an item that occurs in row 1 and event T denote an item that occurs in



column 2. Find P(S or T).

$$P(S\cup T) = 1 - \left[\frac{B+C+H+I}{n}\right]$$

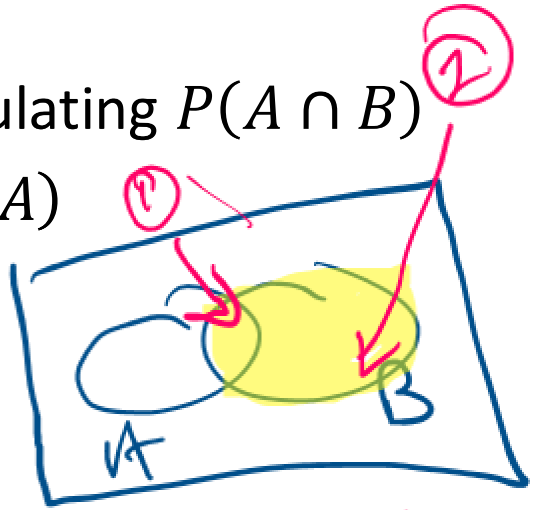
$$P(S\cup T) = \frac{A+D+G+E+F}{n} = \frac{1666+302+29+1+1}{2000}$$

$$= \underline{.9995}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Multiplication Rules

- This rule provides another method for calculating $P(A \cap B)$
- $P(A \cap B) \quad = \quad P(A|B)P(B) = P(B|A)P(A)$
- This leads to the total probability rule



$$\begin{aligned} P(B) &= P(B \cap A) + P(B \cap A') \\ &= P(B|A)P(A) + P(B|A')P(A') \end{aligned}$$

B - 2nd Card Ace, A - 1st Card Ace

- Consider problems from 3rd edition (next slide) and 2-129

P(break | small) = .02

Example Problem 2-76

P(break | large) = .01   P(break) = ?

E 2-76. Samples of laboratory glass are in small, light packaging or heavy, large packaging. Suppose that 2 and 1% of the sample shipped in small and large packages, respectively, break during transit. If 60% of the samples are shipped in large packages and 40% are shipped in small packages, what proportion of samples break during shipment?

P(break) = P(break | small) P(small) + P(break | l) P(large)

problem 2-76

$= .02(.40) + .01(.6) = .014$

Independent Events

- Two events are independent if any one of the following is true:

if one is true, all are true

  1. $P(A|B) = P(A)$
  2. $P(B|A) = P(B)$
  3. $P(A \cap B) = P(A)P(B)$

- Consider problem 2-146

	A	A'
B	70	9
B'	16	5

$$P(B|A) = P(B)$$

$$P(B) = \frac{79}{100} = .79$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{70/100}{86/100} = .81395$$

is $P(B) = P(B|A)$?

no $\rightarrow$ not independent

Reliability Analysis

assume independent events

- Reliability is the application of statistics and probability to determine the probability that a system will be working properly
- Components can be arranged in series. All components must work for the system to work.

$$P(\text{system works}) = P(A \text{ works})P(B \text{ works})$$

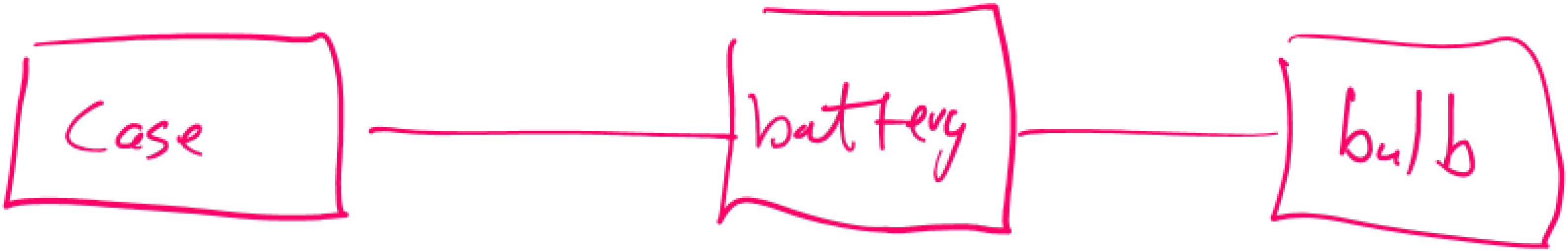
- Components can be arranged in parallel. As long as one component works, the system works.

$$P(\text{system works}) = 1 - \big(1 - P(A \text{ works})\big) \times 1 - P(B \text{ works}))$$

- Consider problem 2-157

Series – most common

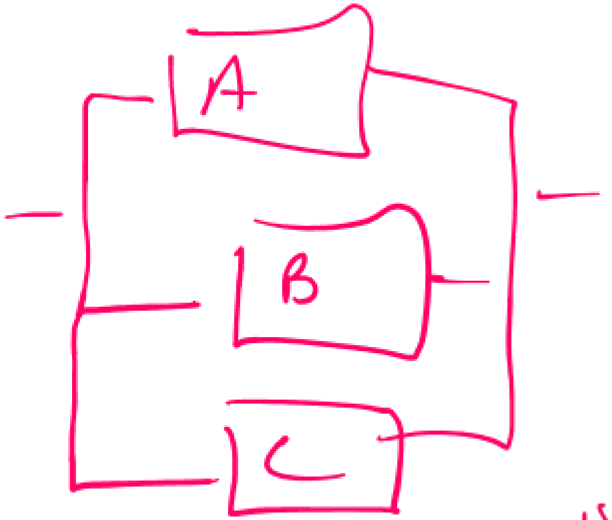
flash light



$P(\text{series}) = P(\text{case})\,P(\text{battery})\,P(\text{bulb})$

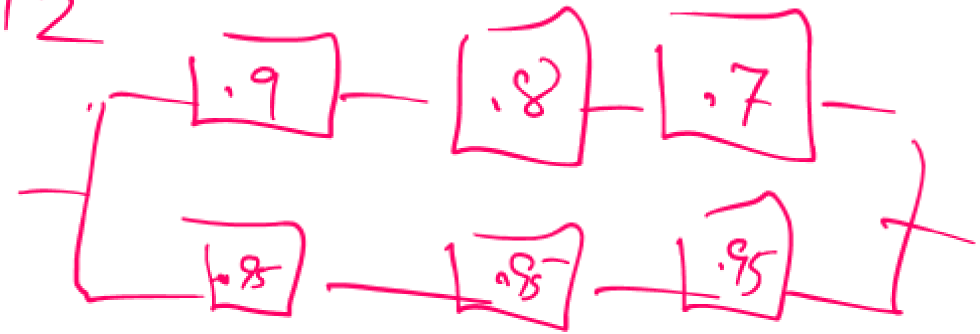
Parallel System

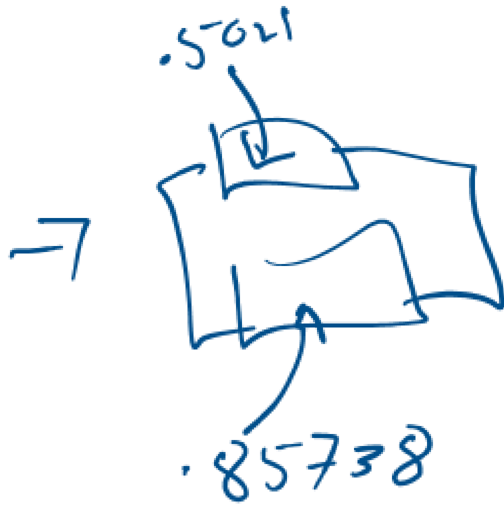
- works as long as 1 or more components works



$$P(\text{parallel}) = 1 - [(1 - P(A))(1 - P(B))(1 - P(C))]$$

P 2.7.12





$P(\text{Top}) = .9(.8)(.7) = .504$

$P(\text{bottom}) = (.95)^3 = .85738$

$$P(\text{system}) = 1 - [(1-.504)(1-.85738)]$$

$$= 1 - .07074 = \underline{.92926}$$