MANE 3332.05

LECTURE 4

Agenda

- Continue Chapter 2 Lecture
- Discuss Easy/Hard
- Single Event Quiz (assigned 9/11/2025, due 9/15/2025)

Handouts

- Lecture 4 Slides - Powerpoint
- Lecture 4 Slides - Marked (pdf)

Axioms (Rules) of Probability

Probability is a number that is assigned to each member of a collection of events from a random experiment that satisfies the following properties:

If S is the sample space and E is any event in a random experiment,

$P(T') = 1 - P(T)$

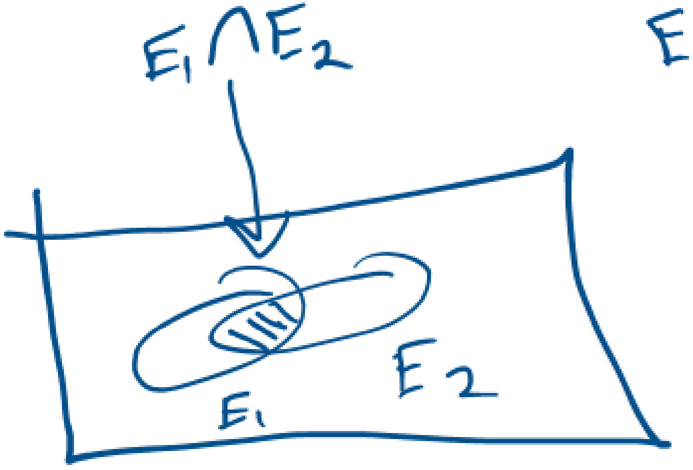
1. $P(S) = 1$
2. $0 \leq P(E) \leq 1$
3. For two events E_1 and E_2 with $E_1 \cap E_2 = \emptyset$

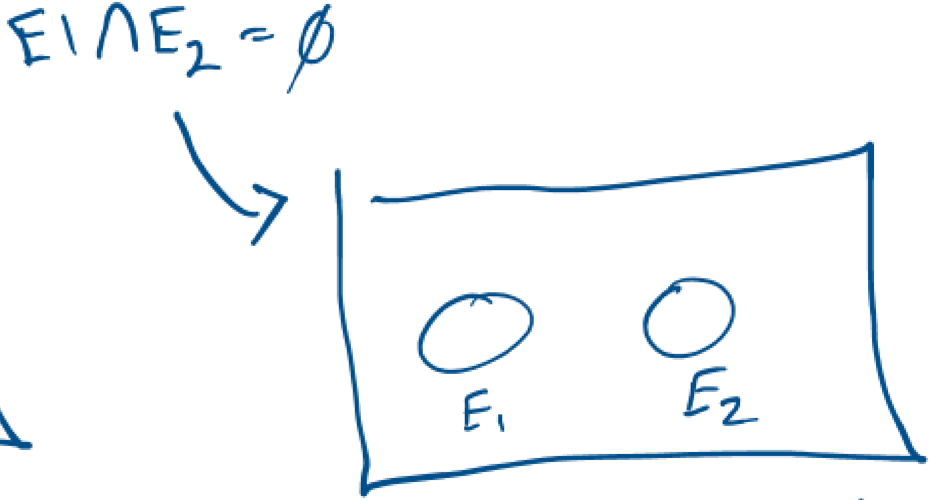
E_1 intersect E_2 is the null or empty set

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

union

- Consider problem 2-70





$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

general rule

mutually exclusive

~~P(A~~

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

special case

Practice Problems - Single Event

A Word of Warning

- It usually looks very easy when I work a problem
- I have been using statistics for almost 40 years
- This is something you MUST practice
- Rework class room examples and textbook examples

Probability of Multiple Events

Cap A

Intersection:

$P(A \cap B)$ is "the probability of A and B occurring (both have to be true)

Union:

$P(A \cup B)$ is "the probability of A or B (or both)"

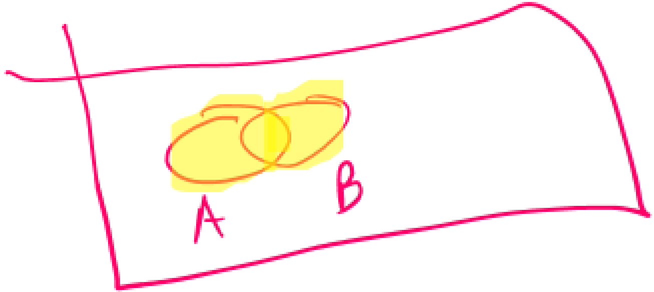
cup

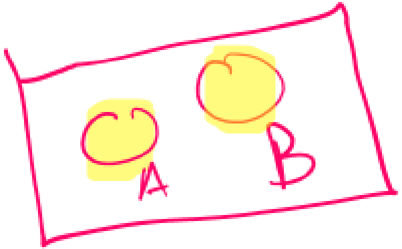
Complement:

$P(A')$ is "the probability of not A"

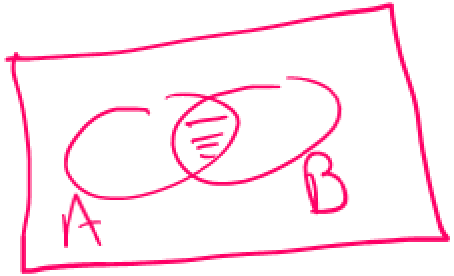
- Venn diagrams are a very useful tool for understanding multiple events and calculating probabilities

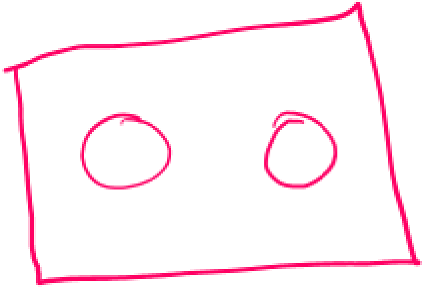
$A \cup B$



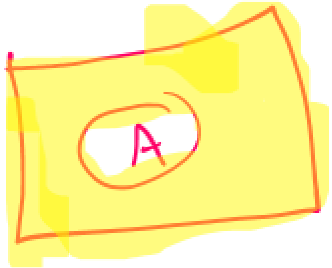


$A \cap B$





A'



Problem 2.3.8

		Shock high	Shock low
Scratch	high	70	9
	low	16	5

$$P(A) = \frac{70+16}{100} = .86$$

$$P(B) = \frac{70+9}{100} = .79$$

$$P(A \cap B) = .70$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= .86 + .79 - .7 = .95$$

	A	A'
B	70	9
B'	16	5

$$P(A') = 1 - P(A) = 1 - .86 = .14$$

$$P(A' \cup B) = \frac{70+9+5}{100} = .8\cancel{6} = .84$$

$$= P(A') + P(B) - P(A' \cap B)$$

$$= .14 + .79 - .09 = .84$$

	A	A'
B	70	9
B'	16	5

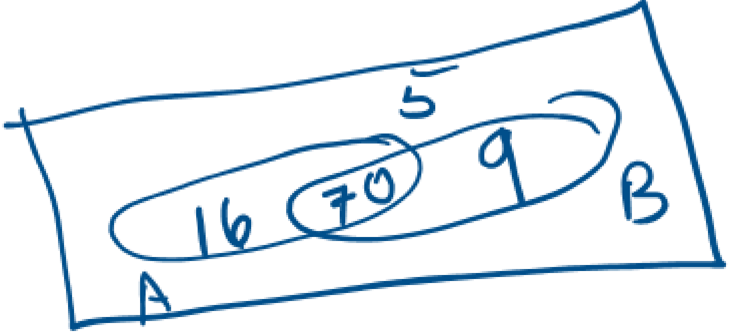
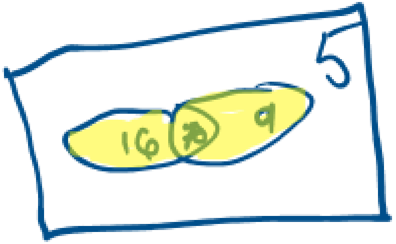


diagram not scale

$P(A \cup B)$



$P(A \cap B)$

Addition Rules

- Used to calculate the union of two events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

general rule

- If two events are mutually exclusive $(A \cap B = \emptyset)$

$$P(A \cup B) = P(A) + P(B)$$

special case

- Consider problems 2-82 and 2-85

Addition Rule for 3 or More Events

- For three events

$$P(A \cup B \cup C) \quad = \quad P(A) + P(B) + P(C)$$
$$-P(A \cap B) - P(A \cap C) - P(B \cap C)$$
$$+P(A \cap B \cap C)$$

- For a set of events to mutually exclusive all pairs of variables must satisfy $E_1 \cap E_2 = \emptyset$
- For a collection of mutually exclusive events,

$$P(E_1 \cup E_2 \cup \cdots \cup E_k) = P(E_1) + P(E_2) + \cdots + P(E_k)$$

$$P(A \cup B \cup C \cup D)$$

$$= P(A) + P(B) + P(C) + P(D)$$

$$- P(A \cap B) - P(A \cap C) - P(A \cap D)$$

$$- P(B \cap C) - P(B \cap D) - P(C \cap D)$$

$$+ P(A \cap B \cap C) + P(A \cap B \cap D)$$

$$+ P(A \cap C \cap D) + P(B \cap C \cap D)$$

$$- P(A \cap B \cap C \cap D)$$

$$\binom{4}{1} = 4$$

$$\binom{4}{2} = 6$$

$$\binom{4}{3} = 4$$

$$\binom{4}{4} = 1$$

Conditional Probability

Let B - ~~probability~~ 2nd card is Ace

- Hayter (2002) states that “For experiments with two or more events of interest, attention is often directed not only at the probabilities of individual events but also at the probability of an event occurring **conditional** on the knowledge that another event has occurred.”

A - 1st card is A

- The **conditional probability** of an event B given an event A, denoted $P(B|A)$ is

given

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$P(B|A) = 3/51$

for $P(A) > 0$

- Consider problems 2-99

	A	
B	70	9
	16	5

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$= \frac{.70}{.86}$$

$$= \underline{.81395}$$

Multiplication Rules

- This rule provides another method for calculating $P(A \cap B)$
- $P(A \cap B) \quad = \quad P(A|B)P(B) = P(B|A)P(A)$
- This leads to the total probability rule
- $$\begin{aligned} P(B) \quad &= \quad P(B \cap A) + P(B \cap A') \\ &= \quad P(B|A)P(A) + P(B|A')P(A') \end{aligned}$$
- Consider problems from 3rd edition (next slide) and 2-129

Example Problem 2-76

E 2-76. Samples of laboratory glass are in small, light packaging or heavy, large packaging. Suppose that 2 and 1% of the sample shipped in small and large packages, respectively, break during transit. If 60% of the samples are shipped in large packages and 40% are shipped in small packages, what proportion of samples break during shipment?

problem 2-76

Independent Events

- Two events are independent if any one of the following is true:
  1. $P(A|B) = P(A)$
  2. $P(B|A) = P(B)$
  3. $P(A \cap B) = P(A)P(B)$
- Consider problem 2-146

Reliability Analysis

- Reliability is the application of statistics and probability to determine the probability that a system will be working properly
- Components can be arranged in series. All components must work for the system to work.

$$P(\text{system works}) = P(A\text{ works})P(B\text{ works})$$

- Components can be arranged in parallel. As long as one component works, the system works.

$$P(\text{system works}) = 1 - \big(1 - P(A\text{ works})\big) \times 1 - P(B\text{ works}))$$

- Consider problem 2-157