MANE 3332.05

LECTURE 2

Agenda

- Re-examine Brightspace
- Questions?
- Continue Day One/Chapter One slides
- Start Chapter 2 lecture, time permitting

Handouts

- Chapter 1 Slides - Powerpoint
- Chapter 1 Slides - Marked Day 2
- Lecture 2 Slides - Powerpoint
- Chapter 2 Slides - Marked Day 2

Chapter 2

Our goal is to understand, quantify, and model the type of variations we encounter. When we incorporate the variation into our thinking and analyses, we can make the informed judgments from our results that are not invalidated by the variation.

Fundamental Definitions

Random experiment

is an experiment that can result in different outcomes, even though it is repeated in the same manner

Sample Space

is the set of all possible outcomes of a random experiment

Event

is a subset of the sample space of a random experiment

Experiment: shuffle deck of cards, flip top card, record color

Events: $\{R\}$, $\{B\}$

Sample Space: $\{\{R\}, \{B\}\}$

Experiment: flip 2 coins, record results

Events: {HT}, {HH}, {TT}, {TH}

probability of HT

$P(HT) = \frac{1}{4}$ → 2 Assumptions

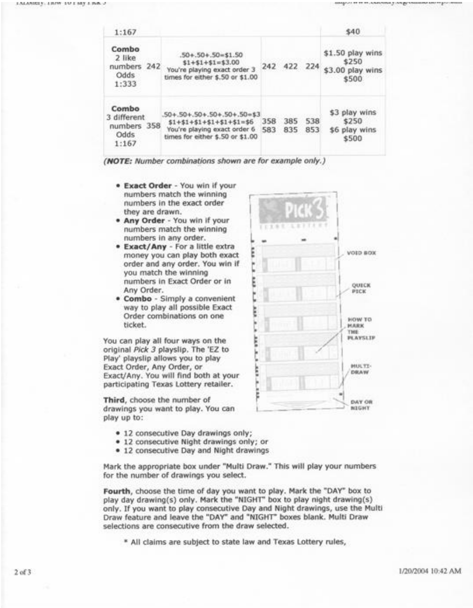
1) Coin flips are independent

2) $P(H) = 1/2$

Examples of Random Experiments, Sample Space, Events

Stopped

- Consider the bead bowl
- Consider the Texas Lottery's Pick Three game (I am not encouraging gambling)



1:167						$40
Combo 2 like numbers Odds 1:333	242	.50+.50+.50=$1.50 $1+$1+$1=$3.00 You're playing exact order 3 times for either $.50 or $1.00	242	422	224	$1.50 play wins $250 $3.00 play wins $500
Combo 3 different numbers Odds 1:167	358	.50+.50+.50+.50+.50+.50=$3 $1+$1+$1+$1+$1+$1=$6 You're playing exact order 6 times for either $.50 or $1.00	358 583	385 835	538 853	$3 play wins $250 $6 play wins $500

*(**NOTE:** Number combinations shown are for example only.)*

- **Exact Order** - You win if your numbers match the winning numbers in the exact order they are drawn.
- **Any Order** - You win if your numbers match the winning numbers in any order.
- **Exact/Any** - For a little extra money you can play both exact order and any order. You win if you match the winning numbers in Exact Order or in Any Order.
- **Combo** - Simply a convenient way to play all possible Exact Order combinations on one ticket.

You can play all four ways on the original *Pick 3* playslip. The 'EZ to Play' playslip allows you to play Exact Order, Any Order, or Exact/Any. You will find both at your participating Texas Lottery retailer.

Third, choose the number of drawings you want to play. You can play up to:

- 12 consecutive Day drawings only;
- 12 consecutive Night drawings only; or
- 12 consecutive Day and Night drawings

Mark the appropriate box under "Multi Draw." This will play your numbers for the number of drawings you select.

Fourth, choose the time of day you want to play. Mark the "DAY" box to play day drawing(s) only. Mark the "NIGHT" box to play night drawing(s) only. If you want to play consecutive Day and Night drawings, use the Multi Draw feature and leave the "DAY" and "NIGHT" boxes blank. Multi Draw selections are consecutive from the draw selected.

* All claims are subject to state law and Texas Lottery rules,

Pick 3 Back Page

Pick 3 Front Page

Tree Diagrams

- Tree diagrams are a useful tool for understanding sample spaces and events. Apply to Pick Three game.

Probability

- The probability of an event is the likelihood that it occurs
- Probability is expressed as a number between 0 and 1
- Probability of an event can be found by dividing the number of outcomes of the desired events divided by the total number of outcomes in the sample space (if all events are equally likely)

Counting Techniques

- Consider ordered versus unordered subsets
- Ordered subsets (Permutations)

$$P_r^n = \frac{n!}{(n-r)!}$$

- Unordered subsets (Combinations)

$$C_r^n = \frac{n!}{r!\,(n-r)!}$$

- Good idea to do a calculator check

Axioms (Rules) of Probability

Probability is a number that is assigned to each member of a collection of events from a random experiment that satisfies the following properties:

If S is the sample space and E is any event in a random experiment,

1. $P(S) = 1$
2. $0 \leq P(E) \leq 1$
3. For two events E_1 and E_2 with $E_1 \cap E_2 = \emptyset$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

- Consider problem 2-70

Practice Problems - Single Event

A Word of Warning

- It usually looks very easy when I work a problem
- I have been using statistics for almost 40 years
- This is something you MUST practice
- Rework class room examples and textbook examples

Probability of Multiple Events

Intersection:

$P(A \cap B)$ is "the probability of A and B occurring

Union:

$P(A \cup B)$ is "the probability of A or B (or both)"

Complement:

$P(A')$ is "the probability of not A"

- Venn diagrams are a very useful tool for understanding multiple events and calculating probabilities

Addition Rules

- Used to calculate the union of two events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- If two events are mutually exclusive ($A \cap B = \emptyset$)

$$P(A \cup B) = P(A) + P(B)$$

- Consider problems 2-82 and 2-85

Addition Rule for 3 or More Events

- For three events

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

- For a set of events to mutually exclusive all pairs of variables must satisfy $E_1 \cap E_2 = \emptyset$
- For a collection of mutually exclusive events,

$$P(E_1 \cup E_2 \cup \cdots \cup E_k) = P(E_1) + P(E_2) + \cdots + P(E_k)$$

Conditional Probability

- Hayter (2002) states that “For experiments with two or more events of interest, attention is often directed not only at the probabilities of individual events but also at the probability of an event occurring **conditional** on the knowledge that another event has occurred.”
- The **conditional probability** of an event B given an event A, denoted $P(B|A)$ is

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

for $P(A) > 0$

- Consider problems 2-99

Multiplication Rules

- This rule provides another method for calculating $P(A \cap B)$
- $P(A \cap B) \quad = \quad P(A|B)P(B) = P(B|A)P(A)$
- This leads to the total probability rule
- $$\begin{aligned} P(B) &= P(B \cap A) + P(B \cap A') \\ &= P(B|A)P(A) + P(B|A')P(A') \end{aligned}$$
- Consider problems from 3rd edition (next slide) and 2-129

Example Problem 2-76

E 2-76. Samples of laboratory glass are in small, light packaging or heavy, large packaging. Suppose that 2 and 1% of the sample shipped in small and large packages, respectively, break during transit. If 60% of the samples are shipped in large packages and 40% are shipped in small packages, what proportion of samples break during shipment?

problem 2-76

Independent Events

- Two events are independent if any one of the following is true:
  1. $P(A|B) = P(A)$
  2. $P(B|A) = P(B)$
  3. $P(A \cap B) = P(A)P(B)$
- Consider problem 2-146

Reliability Analysis

- Reliability is the application of statistics and probability to determine the probability that a system will be working properly
- Components can be arranged in series. All components must work for the system to work.

$$P(\text{system works}) = P(A \text{ works})P(B \text{ works})$$

- Components can be arranged in parallel. As long as one component works, the system works.

$$P(\text{system works}) = 1 - \big(1 - P(A \text{ works})\big) \times 1 - P(B \text{ works}))$$

- Consider problem 2-157