MANE 3332.05

LECTURE 10

Agenda

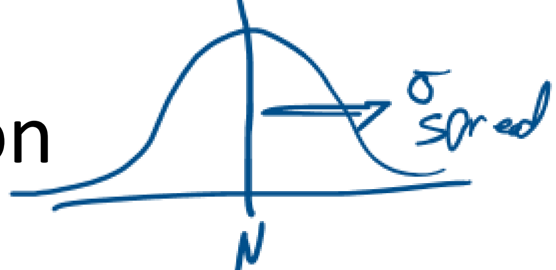
- Continue Chapter 4 lectures
- Poisson Practice Problems (assigned 9/30/2025, due 10/2/2025)
- Poisson Quiz (assigned 10/2/2025, due 10/7/2025)
- Standard Normal Practice Problems (assigned 10/2/2025, due 10/7/2025)
- Schedule

Handouts

- Lecture 10 Slides - Powerpoint
- Lecture 10 Slides - marked (pdf)

Tuesday Date and Topic(s)	Thursday Date and Topic(s)
9/30: Poisson Distribution, Chapter 4	**10/2:** standard normal
10/7: normal distribution	**10/9:** Exponential and Weibull distributions
10/14: Chapter 5 (not on midterm)	**10/16:** Midterm Review
10/21: Midterm Exam	**10/23:** Continue Part Two

The Normal Distribution

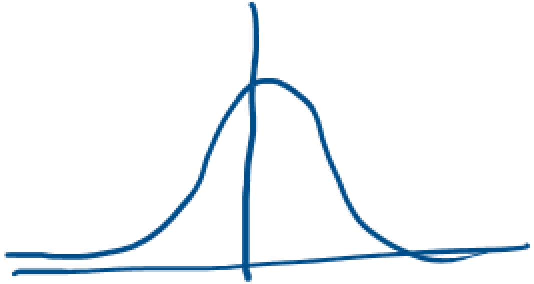


- The normal distribution is the most widely used and important distribution in statistics.
- You must master this!
- A random variable X with probability density function

$$-\infty < x < \infty f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \text{for}$$

has a **normal distribution** with parameters μ and σ where $-\infty < \mu < \infty$ and $\sigma > 0$

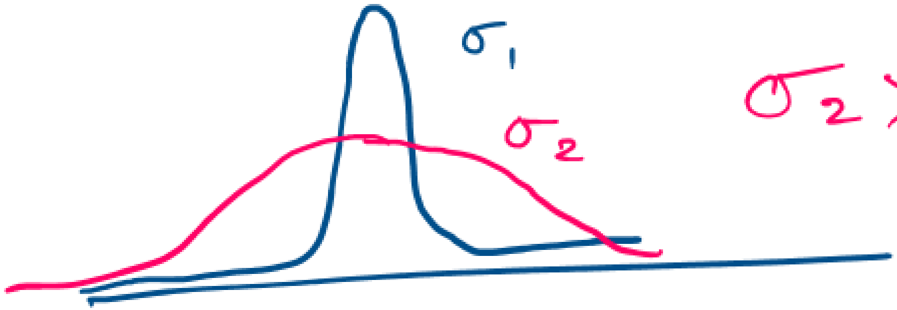
- The normal distribution with parameters μ and σ is denoted $N(\mu, \sigma^2)$
- An interesting web-site is http://www.seeingstatistics.com/seeingTour/normal/shape3.html



$N=5$

σ

$N=15$



$\sigma_2 > \sigma_1$

Mean and Variance of the Normal Distribution

- The mean of the normal distribution with parameters μ and σ is

$$E(X) = \mu$$

- The variance of the normal distribution with parameters μ and σ is

$$V(X) = \sigma^2$$

Central Limit Theorem

- Brief introduction
- States that the distribution of the average of independent random variables will tend towards a normal distribution as n gets large
- More details later

→ $25 \leq n \leq 30$

Calculating Normal Probabilities

- Is somewhat complicated
- The difficulty is $\int_a^b f(x)\,dx$ does not have a closed form solution
- Probabilities must be found by numerical techniques (tabled values)
- It is very helpful to draw a sketch of the desired probabilities (I require this)

μ - mu
σ - sigma
Σ - Sigma
Φ( ) - Phi

The Standard Normal Distribution

- A normal random variable with $\mu = 0$ and $\sigma = 1$ is called a **standard normal** random variable   use letter z
- A standard normal random variable is denoted as z
- The cumulative distribution function for a standard normal is defined to be the function

$$\Phi(z) = P(Z \leq z) = F(z)$$

- These probabilities are contained in Appendix Table III on pages A-8 and A-9

Cumulative Standard Norm

Φ(-3.99) = .000033

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$

TABLE III Cumulative Standard Normal Distribution

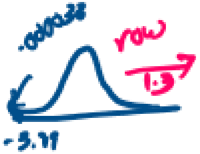
z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	−0.00
−3.9	0.000033	0.000034	0.000036	0.000037	0.000039	0.000041	0.000042	0.000044	0.000046	0.000048
−3.8	0.000050	0.000052	0.000054	0.000057	0.000059	0.000062	0.000064	0.000067	0.000069	0.000072
−3.7	0.000075	0.000078	0.000082	0.000085	0.000088	0.000092	0.000096	0.000100	0.000104	0.000108
−3.6	0.000112	0.000117	0.000121	0.000126	0.000131	0.000136	0.000142	0.000147	0.000153	0.000159
−3.5	0.000165	0.000172	0.000179	0.000185	0.000193	0.000200	0.000208	0.000216	0.000224	0.000233
−3.4	0.000242	0.000251	0.000260	0.000270	0.000280	0.000291	0.000302	0.000313	0.000325	0.000337
−3.3	0.000350	0.000362	0.000376	0.000390	0.000404	0.000419	0.000434	0.000450	0.000467	0.000483
−3.2	0.000501	0.000519	0.000538	0.000557	0.000577	0.000598	0.000619	0.000641	0.000664	0.000687
−3.1	0.000711	0.000736	0.000762	0.000789	0.000816	0.000845	0.000874	0.000904	0.000935	0.000968
−3.0	0.001001	0.001035	0.001070	0.001107	0.001144	0.001183	0.001223	0.001264	0.001306	0.001350
−2.9	0.001395	0.001441	0.001489	0.001538	0.001589	0.001641	0.001695	0.001750	0.001807	0.001866
−2.8	0.001926	0.001988	0.002052	0.002118	0.002186	0.002256	0.002327	0.002401	0.002477	0.002555
−2.7	0.002635	0.002718	0.002803	0.002890	0.002980	0.003072	0.003167	0.003264	0.003364	0.003467
−2.6	0.003573	0.003681	0.003793	0.003907	0.004025	0.004145	0.004269	0.004396	0.004527	0.004661
−2.5	0.004799	0.004940	0.005085	0.005234	0.005386	0.005543	0.005703	0.005868	0.006037	0.006210
−2.4	0.006387	0.006569	0.006756	0.006947	0.007143	0.007344	0.007549	0.007760	0.007976	0.008198
−2.3	0.008424	0.008656	0.008894	0.009137	0.009387	0.009642	0.009903	0.010170	0.010444	0.010724
−2.2	0.011011	0.011304	0.011604	0.011911	0.012224	0.012545	0.012874	0.013209	0.013553	0.013903
−2.1	0.014262	0.014629	0.015003	0.015386	0.015778	0.016177	0.016586	0.017003	0.017429	0.017864
−2.0	0.018309	0.018763	0.019226	0.019699	0.020182	0.020675	0.021178	0.021692	0.022216	0.022750
−1.9	0.023295	0.023852	0.024419	0.024998	0.025588	0.026190	0.026803	0.027429	0.028067	0.028717
−1.8	0.029379	0.030054	0.030742	0.031443	0.032157	0.032884	0.033625	0.034379	0.035148	0.035930
−1.7	0.036727	0.037538	0.038364	0.039204	0.040059	0.040929	0.041815	0.042716	0.043633	0.044565
−1.6	0.045514	0.046479	0.047460	0.048457	0.049471	0.050503	0.051551	0.052616	0.053699	0.054799
−1.5	0.055917	0.057053	0.058208	0.059380	0.060571	0.061780	0.063008	0.064256	0.065522	0.066807
−1.4	0.068112	0.069437	0.070781	0.072145	0.073529	0.074934	0.076359	0.077804	0.079270	0.080757
−1.3	0.082264	0.083793	0.085343	0.086915	0.088508	0.090123	0.091759	0.093418	0.095098	0.096801
−1.2	0.098525	0.100273	0.102042	0.103835	0.105650	0.107488	0.109349	0.111233	0.113140	0.115070
−1.1	0.117023	0.119000	0.121001	0.123024	0.125072	0.127143	0.129238	0.131357	0.133500	0.135666
−1.0	0.137857	0.140071	0.142310	0.144572	0.146859	0.149170	0.151505	0.153864	0.156248	0.158655
−0.9	0.161087	0.163543	0.166023	0.168528	0.171056	[illegible]	[illegible]	0.178786	0.181411	0.184060
−0.8	0.186733	0.189430	0.192150	0.194894	0.197662	0.200454	0.203269	0.206108	0.208970	0.211855
−0.7	0.214764	0.217695	0.220650	0.223627	0.226627	0.229650	0.232695	0.235762	0.238852	0.241964
−0.6	0.245097	0.248252	0.251429	0.254627	0.257846	0.261086	0.264347	0.267629	0.270931	0.274253
−0.5	0.277595	0.280957	0.284339	0.287740	0.291160	0.294599	0.298056	0.301532	0.305026	0.308538
−0.4	0.312067	0.315614	0.319178	0.322758	0.326355	0.329969	0.333598	0.337243	0.340903	0.344578
−0.3	0.348268	0.351973	0.355691	0.359424	0.363169	0.366928	0.370700	0.374484	0.378281	0.382089
−0.2	0.385908	0.389739	0.393580	0.397432	0.401294	0.405165	0.409046	0.412936	0.416834	0.420740
−0.1	0.424655	0.428576	0.432505	0.436441	0.440382	0.444330	0.448283	[illegible]	0.456205	0.460172
0.0	0.464144	0.468119	0.472097	0.476078	0.480061	0.484047	0.488033	0.492022	0.496011	0.500000

row -.2 →

page A-8

Cumulative Standard N[illegible]n

Φ(1.31)



$P(Z < -4.0) \approx 0.0$

$P(Z > 4.0) \approx 1.0$

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\,du$$

TABLE III Cumulative Standard Normal Distribution (continued)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	[illegible]	[illegible]
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	[illegible]	0.999967

page A-9

Φ(3.99) = .999967

Standard

5.1.1 Suppose that $Z \sim N(0, 1)$. Find:

(a) $P(Z \leq 1.34)$

(b) $P(Z \geq -0.22)$

(c) $P(-2.19 \leq Z \leq 0.43)$

(d) $P(0.09 \leq Z \leq 1.76)$

(e) $P(|Z| \leq 0.38)$

(f) The value of x for which $P(Z \leq x) = 0.55$

(g) The value of x for which $P(Z \geq x) = 0.72$

(h) The value of x for which $P(|Z| \leq x) = 0.31$

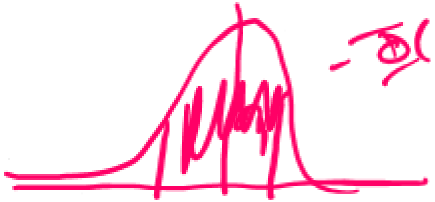
dislike notation

$\mu = 0, \sigma = 1$

Know z, want prob.

Know probability, want z

$\Phi(+.38) - \Phi(-.38)$



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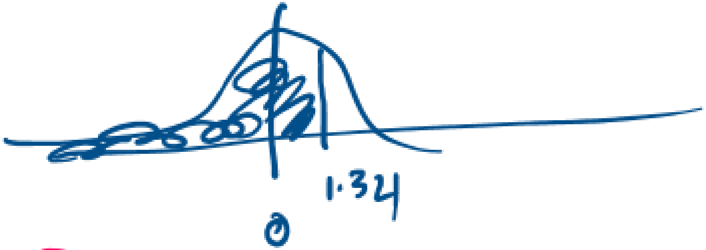
$P(Z < 1.34)$

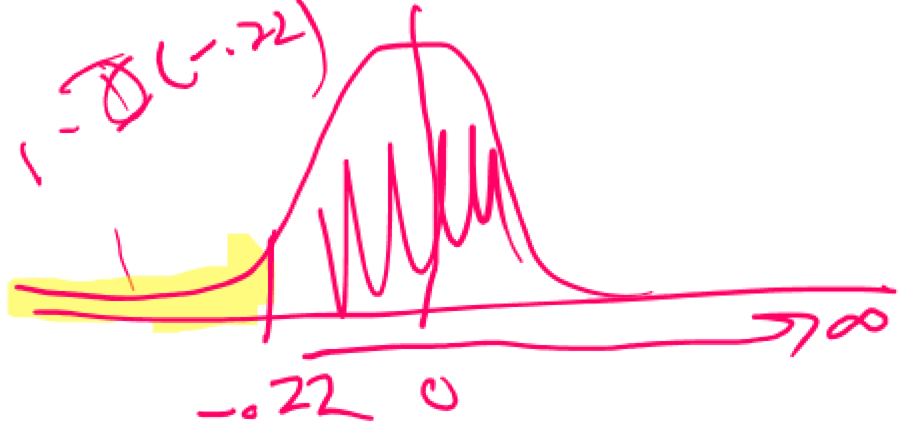
$= \Phi(1.34)$ = .909877

↑ value of z



$P(Z > -.22)$

$= 1 - \Phi(-.22)$



$= 1 - .41294$

$= .58706$

$P(-2.19 < Z < .43)$

$= \Phi(.43) - \Phi(-2.19)$

Find x s.t. $P(Z \le x) = .55$

$X = 0.13$

TABLE III Cumulative Standard Normal Distr

z	0.00	0.01	0.02	0.03
0.0	0.500000	0.503989	0.507978	0.511967
0.1	0.539828	0.543795	0.547758	0.551717
0.2	0.579260	0.583166	0.587064	0.590954
0.3	0.617911	0.621719	0.625516	0.629300

is $Z = \Phi(.12)$ or $\Phi(.13)$ closer to $.55$

$|.547758 - .55|$ or $|.551717 - .55|$

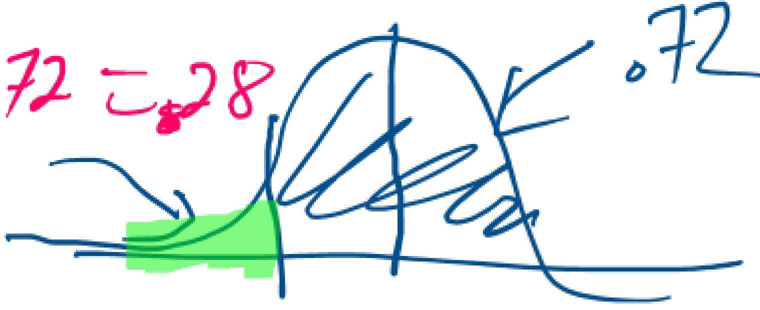
$.00224$ $\qquad$ $.00172$

Find x s.t. $P(z > x) = .72$

$z = -.09 \quad -.08$

−0.7	0.214764	0.217695
−0.6	0.245097	0.248252
−0.5	0.2775[illegible]	0.280957
−0.4	0.312067	0.315614
−0.3	0.348268	0.351973
−0.2	0.385908	0.389739

$1 - .72 = .28$



$x = -.58$

$|.277595 - .281|$ or $|.280957 - .281|$

$.00241 \qquad .000957$

Standard Normal Practice Problems

Question 1 (1 point)

Listen

Let Z be a standard normal random variable, find the value z such that P(Z<z)= 0.0894.

1) 1.55

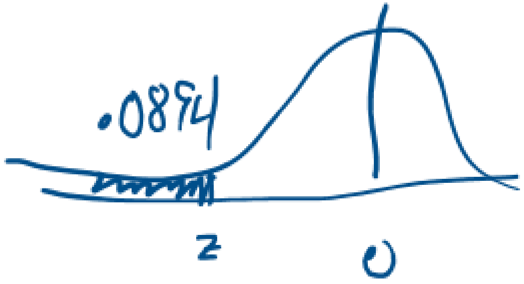
2) -1.34

3) -1.7

4) 1.7

5) 1.34

6) The correct answer is not provided.



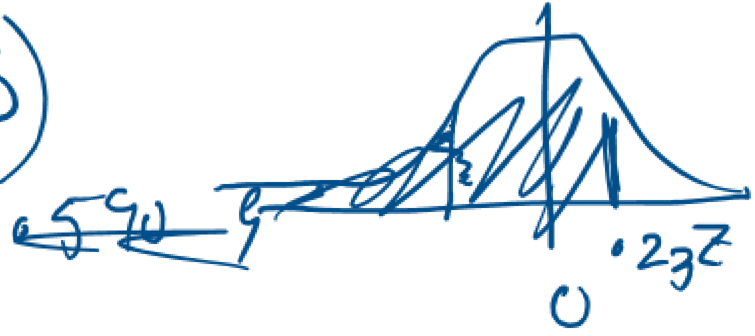
$z = -1.34$

Question 4 (1 point)

Listen

Let Z be a standard normal random variable, find P(Z<0.23).

$= \Phi(.23)$



.590954

1) 0.004848

2) 0.388529

3) 0.164562

4) 0.469046

5) 0.611471

6) 0.365496

7) The correct answer is not provided.

Question 8 (1 point)

Listen

Let Z be a standard normal random variable, find the value z such that P(Z>z)= 0.8486.

1) 3.7

2) 1.03

3) -1.03

4) -0.94

5) The correct answer is not provided.

6) 0.94

$1 - .8486 = .1514$

$z = -1.03$

$\Phi(z) = P(Z < z)$

Attendance 1-A

Normal Probability Problem

5.1.3 Suppose that $X \sim N(10, 2)$. Find:

(a) $P(X \leq 10.34)$

(b) $P(X \geq 11.98)$

(c) $P(7.67 \leq X \leq 9.90)$

(d) $P(10.88 \leq X \leq 13.22)$

(e) $P(|X - 10| \leq 3)$

(f) The value of x for which $P(X \leq x) = 0.81$

(g) The value of x for which $P(X \geq x) = 0.04$

(h) The value of x for which $P(|X - 10| \geq x) = 0.63$

image

Normal Practice Problems

Normal Approximation to the Binomial Distribution

- If X is a binomial random variable,

$$Z = \frac{X - np}{\sqrt{np(1-p)}}$$

is approximately a standard normal random variable. Consequently, probabilities computed from Z can be used to approximate probabilities for X

- Usually holds when

$$np > 5 \quad \text{and } n(1-p) > 5$$

Prob

How good are the approximations?

4. A process yields 10% defective items. If 100 items are randomly selected from the process, what is the probability that the number of defectives

(a) exceeds 13?

(b) is less than 8?

Source: Walpole, Myers, Myers & Ye

image

Normal A

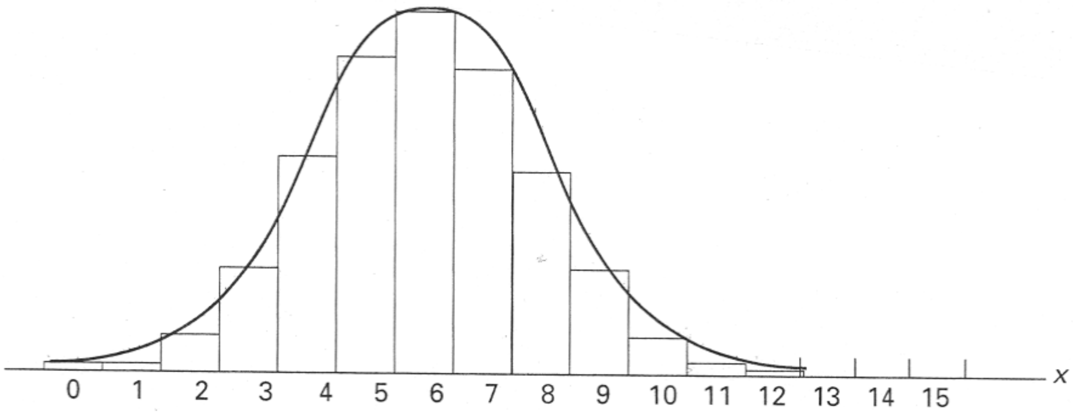


Figure 6.22 Normal approximation of $b(x; 15, 0.4)$.

Source: Walpole, Myers, Myers & Ye

image

Rework Problem using Continuity Correction Factor

- Are the approximations improved?

Normal Approximation to the Poisson Distribution

- If X is a Poisson random variable with $E(X) = \lambda$ and $V(X) = \lambda$,

$$Z = \frac{X - \lambda}{\sqrt{\lambda}}$$

is approximately a standard normal random variable.

Exponential Distribution

- The exponential distribution is widely used in the area of reliability and life-test data.
- Ostle, et. al. (1996) list the following applications of the exponential distribution
  - the number of feet between two consecutive erroneous records on a computer tape,
  - the lifetime of a component of a particular device,
  - the length of a life of a radioactive material and
  - the time to the next customer service call at a service desk

Exponential Distribution

- The PDF for an exponential distribution with parameter $\lambda > 0$ is

$$f(x) = \lambda e^{-\lambda x}, \quad \text{for } 0 \leq x < \infty$$

- The mean of X is

$$\mu = E(X) = \frac{1}{\lambda}$$

- The variance of X is

$$\sigma^2 = V(X) = \frac{1}{\lambda^2}$$

Note that other authors define $f(x) = \frac{1}{\theta} e^{-x/\theta}$. Either definition is acceptable. However one must be aware of which definition is being used.

The Exponential CDF

The CDF for the exponential distribution is easy to derive

$$\begin{aligned} F(x) &= P(X \le x) = \int_{-\infty}^{x} \lambda\, e^{-\lambda y}\, dy \\ &= \int_{0}^{x} \lambda\, e^{-\lambda y}\, dy \\ &= \left(-e^{-\lambda y}\right)\Big|_{y=0}^{x} \\ &= -e^{-\lambda x} - (-e^{0}) \\ &= -e^{-\lambda x} + 1 \\ &= 1 - e^{-\lambda x} \end{aligned}$$

Problem 4-79

4-79. The time to failure (in hours) of fans in a personal computer can be modeled by an exponential distribution with $\lambda = 0.0003$.

(a) What proportion of the fans will last at least 10,000 hours?

(b) What proportion of the fans will last at most 7000 hours?

image

Lack of Memory Property

- The mathematical definition is

$$P(X < t_1 + t_2 | X > t_1) = P(X < t_2)$$

- That is "the probability of a failure time that is less than $t_1 + t_2$ given the failure time is greater than t_1 is the probability that the item's failure time is less than t_2
- This property is unique to the exponential distribution
- Often used to model the reliability of electronic components.

Problem

4-80. The time between the arrival of electronic messages at your computer is exponentially distributed with a mean of two hours.

(a) What is the probability that you do not receive a message during a two-hour period?

(b) If you have not had a message in the last four hours, what is the probability that you do not receive a message in the next two hours?

(c) What is the expected time between your fifth and sixth messages?

image

Relationship to the Poisson Distribution

- Let Y be a Poisson random variable with parameter λ. Note: Y represents the number of occurrences per unit
- Let X be a random variable that records the time between occurrences for the same process as Y
- X has an exponential distribution with parameter λ

Lognormal Distribution

- Let W have a normal distribution with mean θ and variance ω^2; then $X = \exp(W)$ is a **lognormal random variable** with pdf

$$f(x) = \frac{1}{x\omega\sqrt{2\pi}} \exp\left[-\frac{(\ln(x) - \theta)^2}{2\omega^2}\right] \quad 0 < x < \infty$$

- The mean of X is

$$E(X) = e^{\theta + \omega^2/2}$$

- The variance of X is

$$V(X) = e^{2\theta + \omega^2}\left(e^{\omega^2} - 1\right)$$

Example Problem

3-47. Suppose that X has a lognormal distribution with parameters $\theta = 5$ and $\omega^2 = 9$. Determine the following:

(a) $P(X < 13{,}300)$

(b) The value for x such that $P(X \leq x) = 0.95$

(c) The mean and variance of X Montgomery, Runger and Hubble

image

Gamma Distribution

- The random variable X with pdf

$$f(x) = \frac{\lambda^r x^{r-1} e^{-\lambda x}}{\Gamma(r)}, \quad \text{for } x > 0$$

is a **gamma random variable** with parameters $\lambda > 0$ and $r > 0$.

- The gamma function is

$$\Gamma(r) = \int_0^\infty x^{r-1}\, e^{-x}\, dx \quad \text{for } r > 0$$

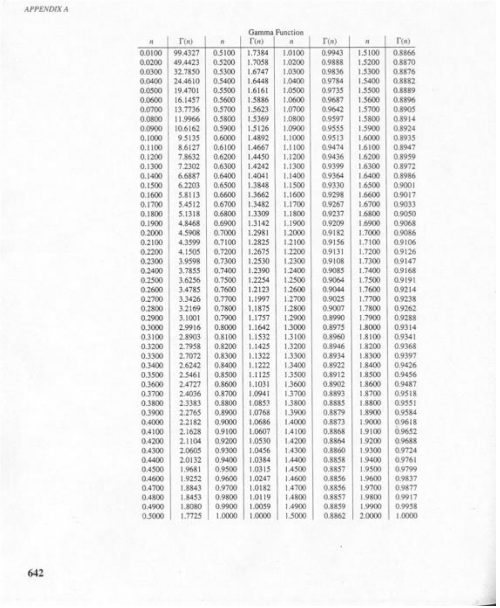
with special properties:

- $\Gamma(r)$ is finite
- $\Gamma(r) = (r-1)\Gamma(r-1)$
- For any positive integer r, $\Gamma(r) = (r-1)!$
  - $\Gamma(1/2) = \pi^{1/2}$

Gamma Distribution

- The mean and variance are
$$\mu = E(X) = r/\lambda \text{ and } \sigma^2 = V(X) = r/\lambda^2$$
- We will not work any probability problems using the gamma distribution

Gamma Tables


APPENDIX A

Gamma Function

n	$\Gamma(n)$	n	$\Gamma(n)$	n	$\Gamma(n)$	n	$\Gamma(n)$
0.0100	99.4327	0.5100	1.7384	1.0100	0.9943	1.5100	0.8866
0.0200	49.4423	0.5200	1.7058	1.0200	0.9888	1.5200	0.8870
0.0300	32.7850	0.5300	1.6747	1.0300	0.9836	1.5300	0.8876
0.0400	24.4610	0.5400	1.6448	1.0400	0.9784	1.5400	0.8882
0.0500	19.4701	0.5500	1.6161	1.0500	0.9735	1.5500	0.8889
0.0600	16.1457	0.5600	1.5886	1.0600	0.9687	1.5600	0.8896
0.0700	13.7736	0.5700	1.5623	1.0700	0.9642	1.5700	0.8905
0.0800	11.9966	0.5800	1.5369	1.0800	0.9597	1.5800	0.8914
0.0900	10.6162	0.5900	1.5126	1.0900	0.9555	1.5900	0.8924
0.1000	9.5135	0.6000	1.4892	1.1000	0.9513	1.6000	0.8935
0.1100	8.6127	0.6100	1.4667	1.1100	0.9474	1.6100	0.8947
0.1200	7.8632	0.6200	1.4450	1.1200	0.9436	1.6200	0.8959
0.1300	7.2302	0.6300	1.4242	1.1300	0.9399	1.6300	0.8972
0.1400	6.6887	0.6400	1.4041	1.1400	0.9364	1.6400	0.8986
0.1500	6.2203	0.6500	1.3848	1.1500	0.9330	1.6500	0.9001
0.1600	5.8113	0.6600	1.3662	1.1600	0.9298	1.6600	0.9017
0.1700	5.4512	0.6700	1.3482	1.1700	0.9267	1.6700	0.9033
0.1800	5.1318	0.6800	1.3309	1.1800	0.9237	1.6800	0.9050
0.1900	4.8468	0.6900	1.3142	1.1900	0.9209	1.6900	0.9068
0.2000	4.5908	0.7000	1.2981	1.2000	0.9182	1.7000	0.9086
0.2100	4.3599	0.7100	1.2825	1.2100	0.9156	1.7100	0.9106
0.2200	4.1505	0.7200	1.2675	1.2200	0.9131	1.7200	0.9126
0.2300	3.9598	0.7300	1.2530	1.2300	0.9108	1.7300	0.9147
0.2400	3.7855	0.7400	1.2390	1.2400	0.9085	1.7400	0.9168
0.2500	3.6256	0.7500	1.2254	1.2500	0.9064	1.7500	0.9191
0.2600	3.4785	0.7600	1.2123	1.2600	0.9044	1.7600	0.9214
0.2700	3.3426	0.7700	1.1997	1.2700	0.9025	1.7700	0.9238
0.2800	3.2169	0.7800	1.1875	1.2800	0.9007	1.7800	0.9262
0.2900	3.1001	0.7900	1.1757	1.2900	0.8990	1.7900	0.9288
0.3000	2.9916	0.8000	1.1642	1.3000	0.8975	1.8000	0.9314
0.3100	2.8903	0.8100	1.1532	1.3100	0.8960	1.8100	0.9341
0.3200	2.7958	0.8200	1.1425	1.3200	0.8946	1.8200	0.9368
0.3300	2.7072	0.8300	1.1322	1.3300	0.8934	1.8300	0.9397
0.3400	2.6242	0.8400	1.1222	1.3400	0.8922	1.8400	0.9426
0.3500	2.5461	0.8500	1.1125	1.3500	0.8912	1.8500	0.9456
0.3600	2.4727	0.8600	1.1031	1.3600	0.8902	1.8600	0.9487
0.3700	2.4036	0.8700	1.0941	1.3700	0.8893	1.8700	0.9518
0.3800	2.3383	0.8800	1.0853	1.3800	0.8885	1.8800	0.9551
0.3900	2.2765	0.8900	1.0768	1.3900	0.8879	1.8900	0.9584
0.4000	2.2182	0.9000	1.0686	1.4000	0.8873	1.9000	0.9618
0.4100	2.1628	0.9100	1.0607	1.4100	0.8868	1.9100	0.9652
0.4200	2.1104	0.9200	1.0530	1.4200	0.8864	1.9200	0.9688
0.4300	2.0605	0.9300	1.0456	1.4300	0.8860	1.9300	0.9724
0.4400	2.0132	0.9400	1.0384	1.4400	0.8858	1.9400	0.9761
0.4500	1.9681	0.9500	1.0315	1.4500	0.8857	1.9500	0.9799
0.4600	1.9252	0.9600	1.0247	1.4600	0.8856	1.9600	0.9837
0.4700	1.8843	0.9700	1.0182	1.4700	0.8856	1.9700	0.9877
0.4800	1.8453	0.9800	1.0119	1.4800	0.8857	1.9800	0.9917
0.4900	1.8080	0.9900	1.0059	1.4900	0.8859	1.9900	0.9958
0.5000	1.7725	1.0000	1.0000	1.5000	0.8862	2.0000	1.0000

642

image

Weibull Distribution

- The random variable X with pdf

$$f(x) = \frac{\beta}{\delta}\left(\frac{x}{\delta}\right)^{\beta-1} \exp\left[-\left(\frac{x}{\delta}\right)^{\beta}\right], \quad \text{for } x > 0$$

is a **Weibull random variable** with scale parameter $\delta > 0$ and shape parameter $\beta > 0$

- The CDF for the Weibull distribution is

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\delta}\right)^{\beta}\right]$$

- The mean of the Weibull distribution is

$$\mu = E(X) = \delta\Gamma\left(1 + \frac{1}{\beta}\right)$$

- The variance of the Weibull distribution is

$$\sigma^2 = V(X) = \delta^2\Gamma\left(1 + \frac{2}{\beta}\right) - \delta^2\left[\Gamma\left(1 + \frac{1}{\beta}\right)\right]^2$$

Weibull Problem

45. Suppose that fracture strength (MPa) of silicon nitride braze joints under certain conditions has a Weibull distribution with $\beta = 5$ and $\delta = 125$ (suggested by data in the article "Heat-Resistant Active Brazing of Silicon Nitride: Mechanical Evaluation of Braze Joints," (*Welding J.*, August 1997: 300s–304s).

a. What proportion of such joints have a fracture strength of at most 100? Between 100 and 150?

b. What strength value separates the weakest 50% of all joints from the strongest 50%?

c. What strength value characterizes the weakest 5% of all joints?

image

Weibull Practice Problems