MANE 3332.05

LECTURE 7

Agenda

- Continue Chapter 3 lectures
- Test 1 Handouts
- Two Events Quiz (assigned 9/18/2025, due 9/23/2025)
- CDF Practice Problems (assigned 9/18/2025, due 9/23/2025)
- CDF Quiz (assigned 9/23/2025, due 9/25/2025)
- Binomial Practice Problems (assigned 9/23/2025, due 9/25/2025)

Handouts

- <u>Lecture 7 Slides Powerpoint</u>
- Lecture 7 Slides marked (pdf)

Mean and Variance of a Discrete Random Variable

• The mean or expected value of a random variable (denoted E(X)) is

$$\mu = E(X) = \sum_{i=1}^{N} x_i f(x_i)$$

• The variance of *X* is

$$\sigma^2 = V(X) = E(X - \mu)^2 = \sum_{i=1}^{N} (x_i - \mu)^2 f(x_i) = \sum_{i=1}^{N} x_i^2 f(x_i) - \mu^2$$

The standard deviation of X is

$$\sigma = \sqrt{V(X)}$$

• Fortunately, we won't often use these formulas. Distributions will have defined functions for μ and σ^2

Bernoulli Distribution

The Bernoulli distribution is one of the simplest statistical distributions.

- The Bernoulli distribution is a random variable that can take only two values Success / failure Ø
- Usually the events are labelled 0 and 1
- The distribution is defined by a single parameter p ($0 \le p \le 1$), takes the values 0 and 1 with P(X = 0) = 1 p and P(X = 1) = p
- The mean is

$$\mu = E(X) = p = \sum_{x \in X} f(x) = O(Xxx) + I \cdot P(xx)$$
s
$$= - \sqrt{n(1-n)}$$

The standard deviation is

$$\sigma = \sqrt{p(1-p)}$$

Summary of Common Probability Distribution (Discrete)

A-4 APPENDIX A Statistical Tables and Charts

TABLE I Sum	nary of Common Probability Distributions			
Name	Probability Distribution	Mean	Variance	Section in Book
Discrete	+(x)	ν	02	
Uniform	$\frac{1}{n}, a \le b$	$\frac{(b+a)}{2}$	$\frac{(b-a+1)^2-1}{12}$	3-5
Binomial	$\binom{n}{x} p^x (1-p)^{n-x}$	np	np(1-p)	3-6
	$x=0,1,\dots,n,0\leq p\leq 1$			
Geometric	$(1-p)^{x-1}p$ $x = 1, 2, \dots, 0 \le p \le 1$	1/p	$(1-p)/p^2$	3-7
Negative binomial	$ \begin{pmatrix} x-1 \\ r-1 \end{pmatrix} (1-p)^{x-r} p^r $ $ x = r, r+1, r+2, \dots, 0 \le p \le 1 $	r/p	$r(1-p)/p^2$	3-7
Hypergeometric	$\frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$ $x = \max(0, n-N+K), 1, \dots$		$np(1-p)\left(\frac{N-n}{N-1}\right)$	3-8
	$\min(K, n), K \le N, n \le N$			
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \dots, 0 < \lambda$	λ	λ	3-9

SKIP

Discrete Uniform Distribution

- A random variable X is a discrete uniform $x_1, x_2, ..., x_n$ has equal probability
- The PMF of a discrete uniform is defined to be

$$f(x_i) = \frac{1}{n}$$

• If the discrete uniform random variable is defined on the consecutive integers a, a + 1, ..., b for $a \le b$. The mean is

$$\mu = E(X) = \frac{b+a}{2}$$

and the standard deviation is

$$\sigma = \sqrt{\frac{(b-a+1)^2-1}{12}}$$

Work problem 3.80

Problem 3.80

3-80. The lengths of plate glass parts are measured to the nearest tenth of a millimeter. The lengths are uniformly distributed with values at every tenth of a millimeter starting at 590.0 and continuing through 590.9. Determine the mean and variance of the lengths.

Binomial Distribution Let x be # of heads and important distribution

- A very common and important distribution. See examples on pages
 80
- A **binomial** experiment is an experiment consisting of n repeated trials such that
 - 1. the trials are independent
 - 2. each trial results in a Bernoulli outcome
 - 3. the probability of success on each trial, denoted as p, remains constant
- To be a binomial distribution, the sampling must be done with replacement. In some situations, the binomial distribution can be used when the sampling is done without replacement

Binomial Distribution

The binomial PMF is

inomial Distribution

is

$$f(x) = \binom{n}{x} p^{x} (1-p)^{n-x}$$

probability of the probabi

where $\binom{n}{x} = \frac{n!}{x!(n-x)!}$

The mean of a binomial random variable is

$$\mu = E(X) = np$$

The standard deviation of X is

$$\sigma = \sqrt{np(1-p)}$$

Example Problem

Source: Montgomery, Runger, Hubele (2004). Engineering Statistics.

- Sketch the probability mass function of X.
- (b) Sketch the cumulative distribution.
- (c) What value of X is most likely?
- (d) What value(s) of X is (are) least likely?
- 3-79. The random variable X has a binomial distribution with n = 20 and p = 0.5. Determine the following probabilities.
- (a) P(X = 15)
- (b) $P(X \le 12)$

- (c) $P(X \ge 19)$ (d) $P(13 \le X < 15)$
- (e) Sketch the cumulative distribution function.

image

$$P(x=15) = +(15)$$

$$= (x) p^{x} (1-p)^{x}$$

$$= (x) p^{x} (1-p)^{x}$$

$$= (x)P(11P)$$

$$= (20)(12)^{15}(1-12)^{20-15}$$

$$= (30)(12)^{15}(1-12)^{20-15}$$

$$= (30)(12)^{15}(1-12)^{20-15}$$

- .01479

(20)

$$P(X \le 12) = f(\emptyset) + f(i) + ... + f(2)$$

$$= F(2)$$

$$= 1 - g f(3) + f(H) + ... + f(20) g$$

$$\Gamma(R) = \binom{20}{19} \binom{12}{9} \binom{1-12}{5}^{20-19} = \frac{.00002}{9.5367450^{-7}}$$

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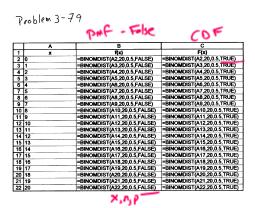
$$= .00002$$

$$P(13 \le x < 15) = F(14) - F(12)$$

$$= F(13) + F(14)$$

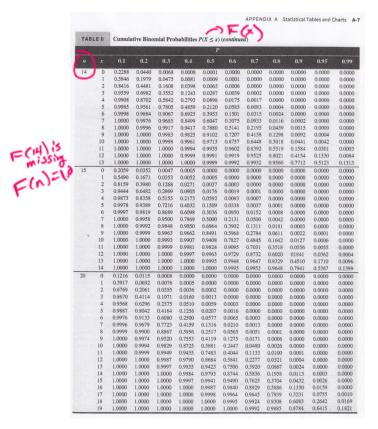
$$= F(13) + F(14)$$

Excel Formula for Binomial Example



image

Cumulative Binomial Probability Tables



Cumulative Binomial Probability Tables

Binomial Practice Problems

Question 1 (1 point)



Let X be a binomial random variable with with parameters: n=3 and p=0.99. Find P(X<1).

O 1) 0.9997



- 3) The correct answer is not provided.
- 4) 0.0003
- O 5) 0.243
- O 6) 1.0



Question 3 (1 point)

◀) Listen ▶

Let X be a binomial random variable with parameters: n=11 and p=0.99. Find P(X=3).

1) The correct answer is not provided.

- O 2) 1.0
- 3) 0.9997
- 4) 0.3138
- O 5) 0.8389



P(x=3)=F(3)-F(2)

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Let X be a binomial random variable with with parameters: n=15 and p=0.9. Find P(X<=4).

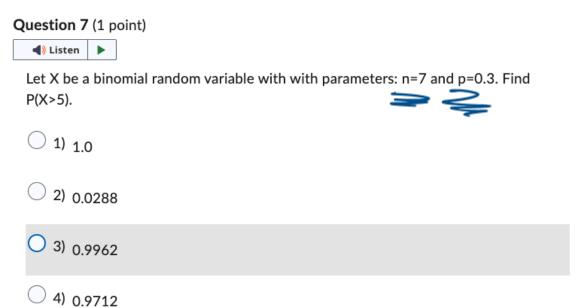
1) 0.2186

N= 15, P=.9

- O 2) 0.2131
- O 3) 0.9978
- O 4) 0.5968
- 5) The correct answer is not provided.
- O 6) 1.0

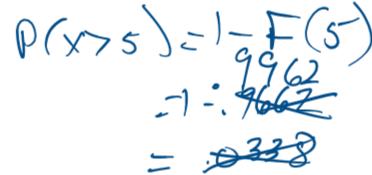
$$P(X \leq 4) = F(4)$$

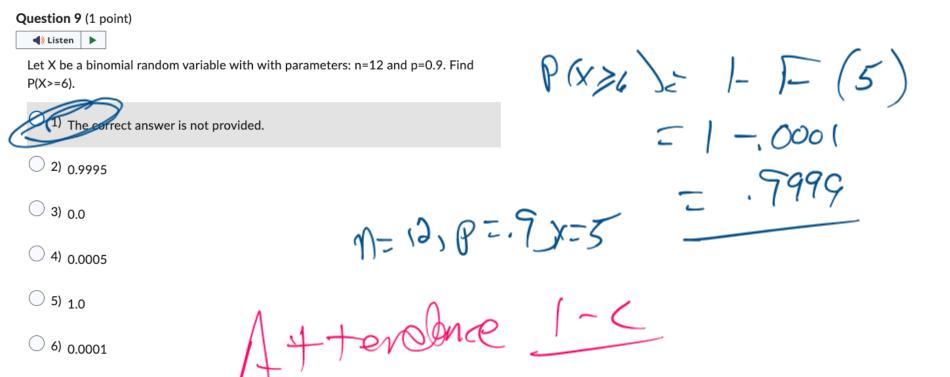
$$= 0.0$$



5) The correct answer is not provided.

6) 0.025





Hypergeometric Distribution

The hypergeometric distribution is one of the commonly occurring distributions in quality.

- A random variable is hypergeometric when a set of N objects contains
 - K objects classified as successes and
 - -N-K objects classified as failures
 - a sample of size n is selected **without replacement** from the N objects, where $K \leq N$ and $n \leq N$

Hypergeometric Distribution

• The hypergeometric PMF is

$$f(x) = \frac{\binom{K}{x} \binom{N - K}{n - x}}{\binom{N}{n}}$$

• The mean of X is

$$E(X) = \mu = np$$

The variance of X is

$$\sigma^2 = V(X) = np(1-p) \left[\frac{N-n}{N-1} \right]$$

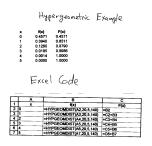
Hypergeometric Example Problem

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Excel for Hypergeometric Example



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Binomial Approximation to the Hypergeometric Distribution

 The mean and variance of the hypergeometric and binomial distribution are very similar. The variance only differs by the finite population correction factor,

$$\frac{N-n}{N-1}$$

- Sampling with replacement is equivalent to sampling from an infinite set (without replacement) because the proportion remains constant
- If *n* is small relative to *N*, then the finite correction is negligible and the binomial distribution can be used as an approximation to the hypergeometric.
- A rule of thumb is to use this approximation when N/n > 20.

Geometric Distribution

- Montgomery and Runger (2003) define a geometric random variable to be the number of trials until the first success of a series of independent Bernoulli trials, with constant probability p of success
- The PMF of a geometric distribution is

$$f(x) = (1-p)^{x-1}p, x = 1,2,...$$

The mean of a geometric random variable is

$$\mu = E(X) = \frac{1}{p}$$

The variance of a geometric random variable is

$$\sigma^2 = V(X) = \frac{1-p}{p^2}$$

Geometric Distribution Example

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Geometric Distribution Example

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Negative Binomial Distribution

- Montgomery and Runger (2003) define a negative binomial random variable to be the number of trials until r successes are observed of a series of independent Bernoulli trials, with constant probability p of success
- The geometric distribution is a special case of the negative binomial distribution with r=1
- The PMF of a negative binomial distribution is

$$f(x) = {x-1 \choose r-1} (1-p)^{x-r} p^r, \ x = r, r+1, ...$$

• The mean of a negative binomial random variable is

$$\mu = E(X) = \frac{r}{p}$$

• The variance of a negative binomial random variable is

$$\sigma^2 = V(X) = \frac{r(1-p)}{p^2}$$

Negative Binomial Example



Poisson Process

- The number of events over an interval (such as time) is a discrete random variable that is often modelled by the Poisson distribution
- The length of the interval between events is often modeled by the (continuous) exponential distribution
- These two distributions are related

Poisson Process

- The number of events over an interval (such as time) is a discrete random variable that is often modelled by the Poisson distribution
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Poisson Process

Assume that the events occur at random throughout the interval. If the interval can be partitioned into subintervals of small enough length such that

- 1. The probability of more than one count in a subinterval is zero
- 2. The probability of one count in a subinterval is the same for all subintervals and proportional to the length of the subinterval, and
- 3. The count in each subinterval is independent of other subintervals, the random experiment is called a *Poisson process*

Poisson Distribution

If the mean number of counts in the interval is $\lambda > 0$, the random variable X that equals the number of counts in the interval has a **Poisson distribution** with parameter λ

The Poisson PMF is

$$f(x) = \frac{e^{-\lambda} \lambda^x}{x!}, x = 0,1,2,...$$

• The mean of a Poisson random variable is

$$E(X) = \mu = \lambda$$

The variance of a Poisson random variable is

$$V(X) = \sigma^2 = \lambda$$

Poisson Practice Problems

Poisson Example

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Poisson Example

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