MANE 3351

Lecture 12

Classroom Management

Agenda

- Test 1 graded. Will return after lecture and before lab
- Discuss Schedule
- Lecture: Secant Method; last root finding method
- Lab Session: Lab Five - GitHub/GitHub Desktop

Calendar

Week	Monday Lecture	Wednesday Lecture
7	**10/13:** Secant Method	**10/15:** Trapezoid Rule
8	**10/20:** Simpson's Rule	**10/22:** Romberg Integration
9	**10/27:** Gaussian Quadrature	**10/29:** Numerical Differentiation (not on Test 2)
10	**11/3:** Linear Algebra	**11/5:** Test 2 (Root Finding and Numerical Integration)

Resources

Handouts

- Lecture 12 Slides
- Lecture 12 Marked Slides

Lecture 12 Content

- Today's topic is the secant method.
- The secant method does not utilize derivative information as Newton's method does.
- The secant method is also similar to the false position method.
- The secant method is the last root finding method covered

Limit Definition of the Derivative

Recall the limit definition of the derivative[1]

LIMIT DEFINITION OF THE DERIVATIVE

Once we know the most basic differentiation formulas and rules, we compute new derivatives using what we already know. We rarely think back to where the basic formulas and rules originated.

The geometric meaning of the derivative

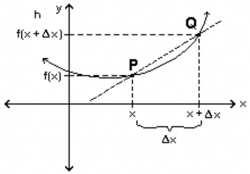
$$f'(x) = \frac{df(x)}{dx}$$

is the slope of the line tangent to $y = f(x)$ at x.

Let's look for this slope at P:

The **secant** line through P and Q has slope

$$\frac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}.$$



We can approximate the **tangent** line through P by moving Q towards P, decreasing Δx. In the limit as $\Delta x \to 0$, we get the tangent line through P with slope

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}.$$

We define

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)^*}{\Delta x}.$$

Definition of Derivative

Geometric Inspiration

Cheney and Kincaid (2004)[2] demonstrate the geometric inspiration for secant method

Requires: 2 Starting values but NOT bracketed

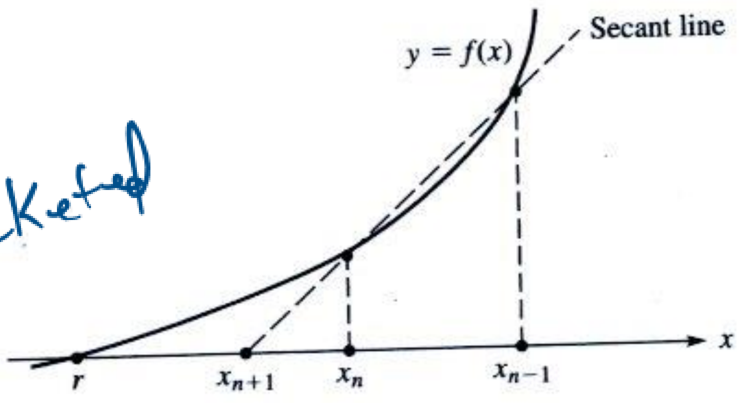


FIGURE 3.6
Secant method

Secant method

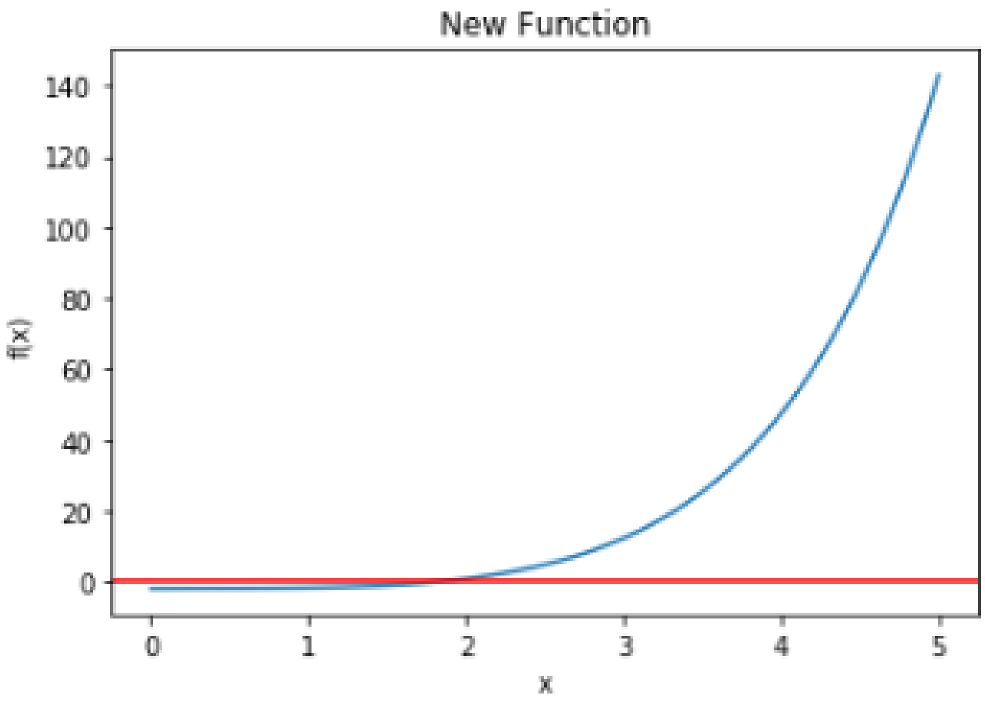
Secant Method

- The formula is simply

$$x_{n+1} = x_n - g(x_n)\frac{x_n - x_{n-1}}{g(x_n) - g(x_{n-1})}$$

Example Problem Used for Newton's Method

Consider a new function, $f(x) = e^x + 2^{-x} + 2\cos(x) - 6 = 0$



Example Function for Newton's method

Pseudo-code

Brin (2020)[3] provides the following pseudo-code

Secant Method (pseudo-code)

A straightforward implementation of the secant method can easily be inefficient due to the number of times g appears in formula on page 67. The pseudo-code below takes great care not to compute each value of g more than once. If it seems more complicated than necessary, this is likely the source of the complication.

Assumptions: g has a root at $\hat{x}$. g is differentiable in a neighborhood of $\hat{x}$. x_0 and x_1 are sufficiently close to $\hat{x}$.

Input: Initial values x_0 and x_1; function g; desired accuracy *tol*; maximum number of iterations N.

Step 1: Set $y_0 = g(x_0)$; $y_1 = g(x_1)$

Step 2: For $j = 1 \ldots N$ do Steps 3-5:

- **Step 3:** Set $x = x_1 - y_1 \frac{x_1 - x_0}{y_1 - y_0}$;
- **Step 4:** If $|x - x_1| \leq tol$ then return x;
- **Step 5:** Set $x_0 = x_1$; $y_0 = y_1$; $x_1 = x$; $y_1 = g(x_1)$

Step 6: Print "Method failed. Maximum iterations exceeded."

Output: Approximation x near exact fixed point, or message of failure.

Secant Method pseud-ocode

Convergence

- Brin (2020)[4] study the performance of Secant Method
- The secant method converges with order $\frac{1+\sqrt{5}}{2} = 1.62$
- Not quite as fast as Newton’s Method (quadratic)

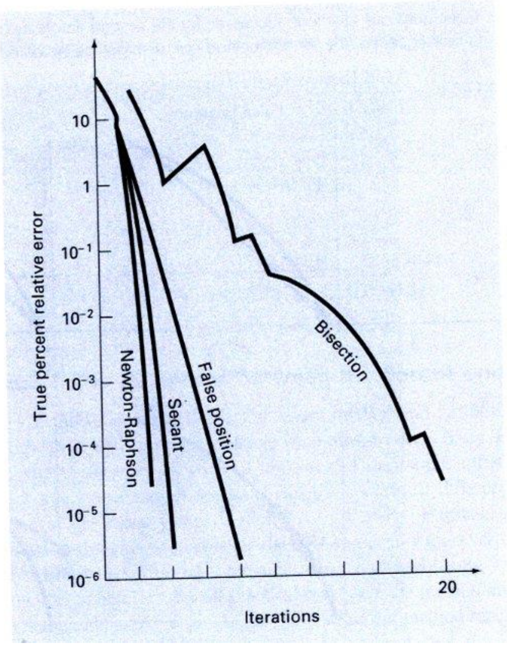
Similarity to False Position

- Secant method: $x_{n+1} = x_n - g(x_n)\frac{x_n - x_{n-1}}{g(x_n) - g(x_{n-1})}$
- False Position method: $x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$
- Secant method is not guaranteed to bracket result when starting

Speed of Convergence

Chapra and Canale (2015)[5] compared the performance of various root finding methods

FIGURE 6.9
Comparison of the true percent relative errors ε_t for the methods to determine the roots of $f(x) = e^{-x} - x$.



Comparison

Secant Code

```
import math
def f(x):
    return (math.exp(x)+2**(-x)+2*math.cos(x)-6)

N=100
tol=0.0005
x0=6.0
x1=5.0
# step 1
y0=f(x0)
y1=f(x1)
# counter is additional
counter=0
for j in range(N+1):
    counter=counter+1
    # step 3
    x=x1-y1*((x1-x0)/(y1-y0))
    if math.fabs(x-x1)<tol:
        print("the root is {} with value {}, required {} steps".format(x,f(x),counter))
        break
    # step 5
    x0=x1
    y0=y1
    x1=x
    y1=f(x1)
print("completed")
```

$x_0 = 6$, $x_1 = 5$, $f(x) = e^x + 2^{-x} + 2\cos(x) - 6$

$y_0 = f(x_0) = f(6) = e^6 + 2^{-6} + 2\cos(6) - 6 = 399.365$

$y_1 = f(x_1) = f(5) = e^5 + 2^{-5} + 2\cos(5) - 6 = 143.012$

$x = x_1 - y_1 \left(\frac{x_1 - x_0}{y_1 - y_0} \right) = 5 - 143.012 \left(\frac{5-6}{143.012 - 399.365} \right)$

$= 4.442$

$|x - x_1| = |5 - 4.442| = .558$ (not less than .0005)

$\Rightarrow$ another iteration

$x_0 = x_1 = 5$

$y_0 = y_1 = 143.012$

$x_1 = x = 4.442$

$y_1 = f(4.442)$

1) Check points

2) Compare to my Solutions

Test 1 22%

$20 \rightarrow -80(.22) = -17.6$ on final grade

Notes

1. https://math.hmc.edu/calculus/hmc-mathematics-calculus-online-tutorials/single-variable-calculus/limit-definition-of-the-derivative/

2. Cheny, W., and Kincaid, D., (2004), *Numerical Mathematics and Computer, 5th edition*

3. Brin, L, (2020), *Tea Time Numerical Analysis: Experiences in Mathematics, 3rd edition*

4. Brin, L, (2020), *Tea Time Numerical Analysis: Experiences in Mathematics, 3rd edition*

5. Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*