MANE 3351

Lecture 13

Classroom Management

Agenda

- Homework 3 Assignment
- Numerical Integration
- Lab 5 if not completed on Monday

Calendar

Week	Monday Lecture	Wednesday Lecture
7	**10/13:** Secant Method	**10/15:** Trapezoid Rule
8	**10/20:** Simpson's Rule	**10/22:** Romberg Integration
9	**10/27:** Gaussian Quadrature	**10/29:** Numerical Differentiation (not on Test 2)
10	**11/3:** Linear Algebra	**11/5:** Test 2 (Root Finding and Numerical Integration)

Resources

Handouts

- Lecture 13 Slides
- Lecture 13 Marked Slides

Lecture 13 Content

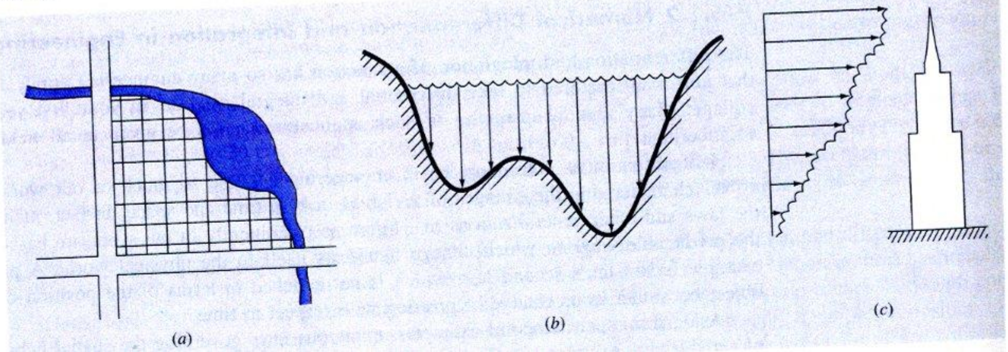
- Today’s topic is numerical integration.
- This is a major new topic after root finding.
- Trapezoid Rule

Introduction to Numerical Integration

In layman's terms, an integral calculates the area under a curve

Frequently used in engineering analysis

FIGURE PT6.8
Examples of how integration is used to evaluate areas in engineering applications. (*a*) A surveyor might need to know the area of a field bounded by a meandering stream and two roads. (*b*) A water-resource engineer might need to know the cross-sectional area of a river. (*c*) A structural engineer might need to determine the net force due to a nonuniform wind blowing against the side of a skyscraper.



Examples of integration

normal

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\int_{-2}^{-1} f(x)\,dx$$

In electrical field theory, it is proved that the magnetic field induced by a current flowing in a circular loop of wire has intensity

$$H(x) = \frac{4Ir}{r^2 - x^2} \int_0^{\pi/2} \left[1 - \left(\frac{x}{r}\right)^2 \sin^2\theta\right]^{1/2} d\theta$$

where I is the current, r the radius of the loop, and x the distance from the center to the point where the magnetic intensity is being computed ($0 \leqq x \leqq r$). If I, r, and x are given, we have a nasty integral to evaluate. It is an **elliptic integral** and not expressible in terms of familiar functions. But H *can* be computed precisely by the methods of this chapter. For example, if $I = 15.3$, $r = 120$, and $x = 84$, we find $H = 1.35566\,1135$ accurate to nine decimals.

Another Integration Example

Definitions

Cheney and Kincaid (2004)[1] provide the following definitions

- **Indefinite integral :** $\int x^2 \ dx = \frac{1}{3}x^3 + C$
- **Definite integral:** $\int x^2 \ dx = \frac{8}{3}$

Numerical Integration

Kiusalaas (2013)[2] suggest three major approaches to numerical integration that we will investigate:

1. Newton-Cotes
   a. Trapezoid rule (n=1)
   b. Simpson's rule (n=2)
   c. 3/8 Simpson's rule (n=3)
2. Romberg Integration
3. Gaussian Quadrature

Note: there are many different techniques for numerical integration than the ones listed above

Newton-Cotes Formulas

Kiusalass (2013)[3] provide the following illustration to explain Newton-Cotes techniques

6.2 Newton-Cotes Formulas

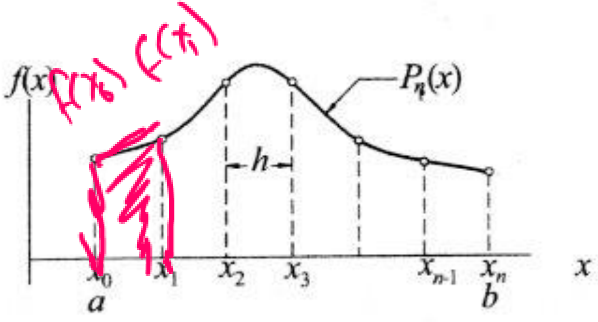
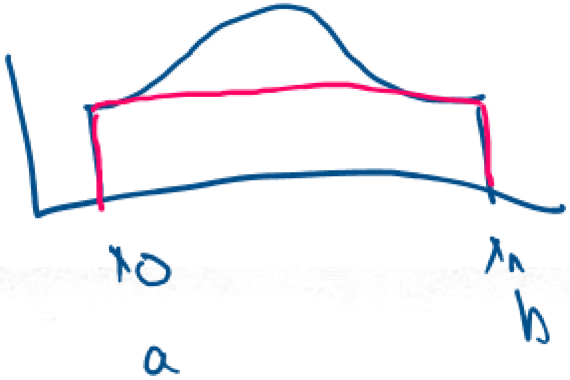
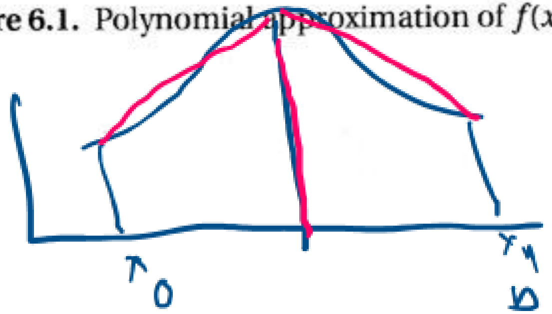


Figure 6.1. Polynomial approximation of $f(x)$.





Newton Cotes Approach

Parabola → 2nd order or x^2

Simpson's (1/3) Rule

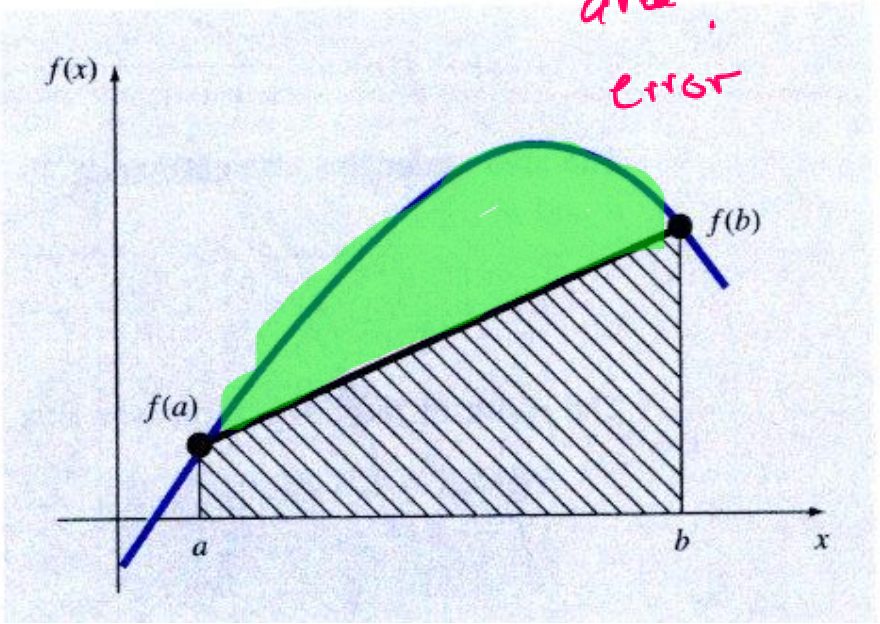
fit 3rd order polynomial

Simpson's (3/8) Rule

Trapezoid Rule

Chapra and Canale (2015)[4] provide the figure shown below illustrating the trapezoid rule

FIGURE 21.4
Graphical depiction of the trapezoidal rule.



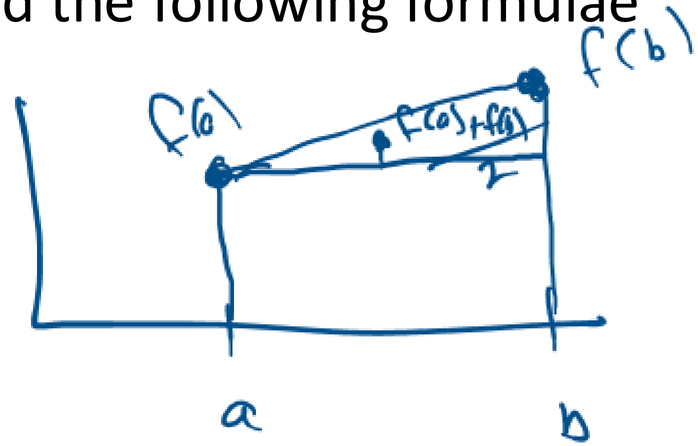
Trapezoid Rule

square = base * height
(b-a) ?

Trapezoid Rule, continued

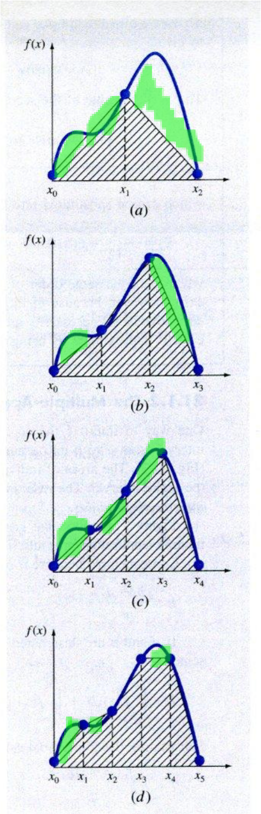
Chapra and Canale (2015)[5] provided the following formulae

- $I = (b-a)\frac{f(a)+f(b)}{2}$
- $E = -\frac{1}{12}f''(\xi)(b-a)^3$



Multiple Applications of the Trapezoid Rule

Typically, the region form a to b is sub-divided into multiple regions and then the Trapezoid Rule for each region is applied. Chapra and Canale (2015)[6] illustrate this concept.



Multiple Trapezoid Rules

Uniform Spacing

Cheney and Kincaid (2004)[7] the following formula for composite (multiple) applications of the Trapezoid Rule

$$\int_a^b = f(x)dx \approx T(f;P) = h\left\{\sum_{i=1}^{n-1} f(x_i) + \frac{1}{2}[f(x_0) + f(x_n)]\right\}$$

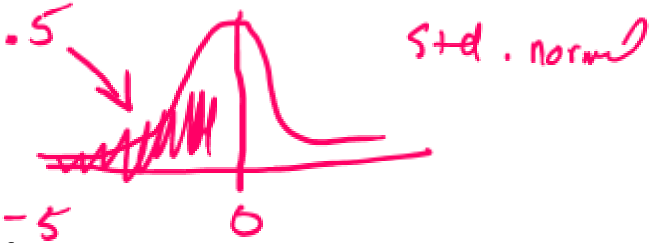
$$h = \frac{b-a}{n}$$

Pseudo-code

Cheney and Kincaid (2004)[8] provided the following pseudo-code for the composite trapezoid rule

```
program Trapezoid
integer parameter n ← 60
real parameter a ← 0, b ← 1
integer i
real h, sum, x
h ← (b − a)/n
sum ← ½[f(a) + f(b)]
for i = 1 to n − 1 do
    x ← a + ih
    sum ← sum + f(x)
end for
sum ← (sum)h
output sum
end Trapezoid
```

Trapezoid Rule Pseudo-code



Python Code for Multiple Trapezoid Rule Applications

```
import math
def f(z):
    return (math.exp(-0.5*z**2)/((2.0*math.pi)**0.5))

n=4
a=-5.0
b=0.0
h=(b-a)/n
sum=0.5*(f(a)+f(b))
for i in range(1,n):
    x=a+i*h
    sum=sum+f(x)
sum=sum*h
print("The area is {} for {} sub-intervals".format(sum,n))
```

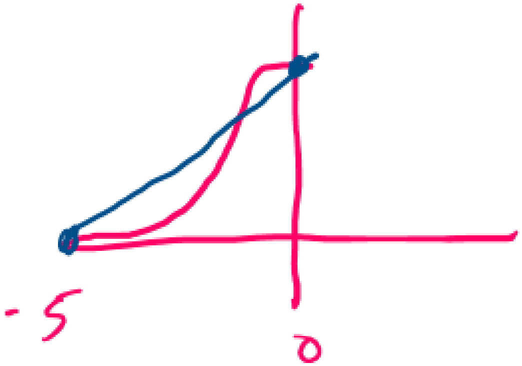
$a = -5, \quad b = 0$

$$I = (b-a)\,\frac{f(a)+f(b)}{2}$$

$$= (0 - -5)\,\frac{f(-5)+f(0)}{2}$$

$$= 5\left(\frac{0+.39894}{2}\right)$$

$$= 0.99735$$



Notes

1. Cheny, W., and Kincaid, D., (2004), *Numerical Mathematics and Computer, 5th edition*

2. Kiusalaas, J. (2013), *Numerical Methods in Engineering with Python 3*

3. Kiusalaas, J. (2013), *Numerical Methods in Engineering with Python 3*

4. Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

5. Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

6. Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

7. Cheny, W., and Kincaid, D., (2004), *Numerical Mathematics and Computer, 5th edition*

8. Cheny, W., and Kincaid, D., (2004), *Numerical Mathematics and Computer, 5th edition*