MANE 3351

Lecture 14

Classroom Management

Agenda

- Simpson’s Rule
- Lab 5 due before 9:30 AM
- Lab 6 Assigned today
- Homework 3 due 10/22/2025 before 11:59 PM

Calendar

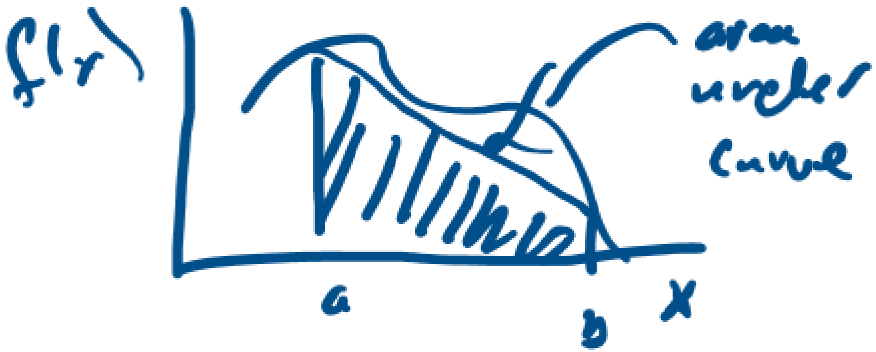
Week	Monday Lecture	Wednesday Lecture
8	**10/20:** Simpson's Rule	**10/22:** Romberg Integration
9	**10/27:** Gaussian Quadrature	**10/29:** Numerical Differentiation (not on Test 2)
10	**11/3:** Linear Algebra	**11/5:** Test 2 (Root Finding and Numerical Integration)

Resources

Handouts

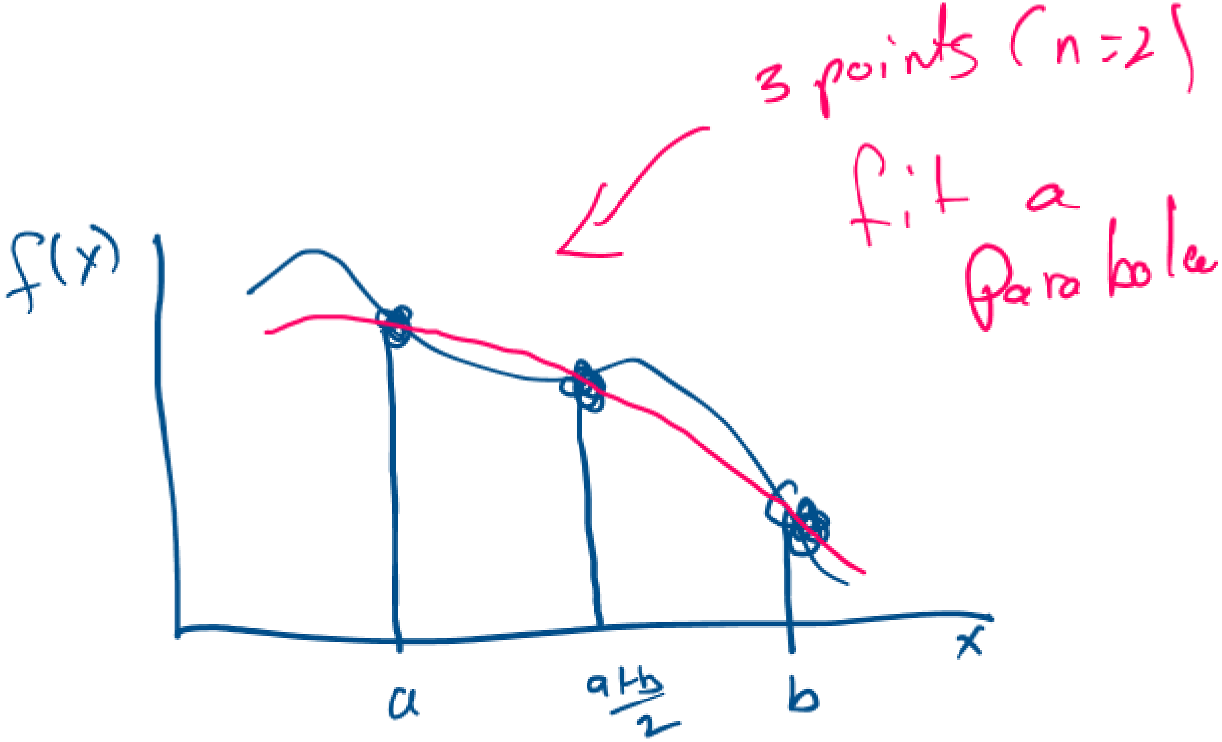
- Lecture 14 Slides
- Lecture 14 Marked Slides

Lecture 14



Today's topic is Simpson's Rule.

- Simpson's (1/3) rule is Newton-Cotes with $n = 2$
- Chapra and Canale (2015)[1] illustrate Simpson's rule in the figure shown below Simpson's Rule


3 points (n=2)
fit a parabola
f(x)
x
a
a+b
2
b

Definition of Simpson's Rule

- Chapra and Canale(2015)[2] provide the following definition of Simpson's rule

$$I = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$$

where $h = (b - a)/2$

- Note that Simpson's rule is of the form

$$I \approx (b-a)\frac{f(x_0) + 4f(x_1) + f(x_2)}{6}$$

$$I \approx \text{width} \times \text{average height}$$

Simpson's Rule Error Analysis

- Chapra and Canale (2015)[3] provides the following definition:

$$I \quad = \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)] - \frac{1}{90}f^{(4)}(\xi)h^5$$

$$I \quad = \text{Simpson's 1/3 approximation} - \text{Truncation error}$$

Trapezoid Rule, continued

Chapra and Canale (2015)[3] provided the following formulae

- $I = (b-a)\frac{f(a)+f(b)}{2}$
- $E = -\frac{1}{12}f''(\xi)(b-a)^3$

Multiple Applications of Simpson's Rule

- The number of segments must be even
- The formula is

$$I \approx (b-a)\frac{f(x_0) + 4\sum_{i=1,3,5}^{n-1} f(x_i) + 2\sum_{j=2,4,6}^{n-2} f(x_j) + f(x_n)}{3n}$$

where $h = \frac{b-a}{n}$

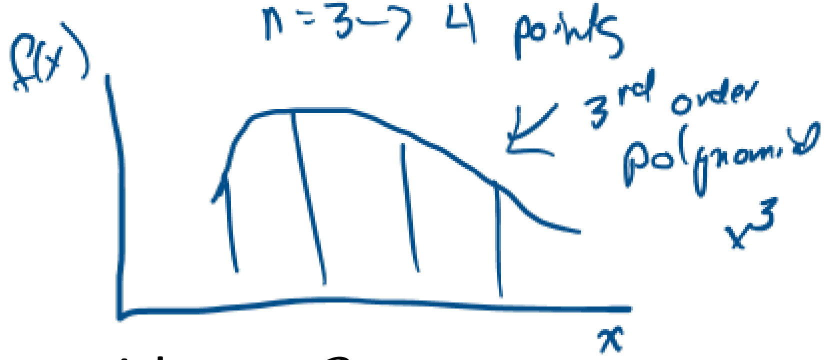
Pseudo-code: Simpson’s 1/3 Rule

- CodeSansar[4] provides the following pseudo-code Simpson’s 1/3 pseudo-code

Python Code for Multiple Simpson’s 1/3 Rule

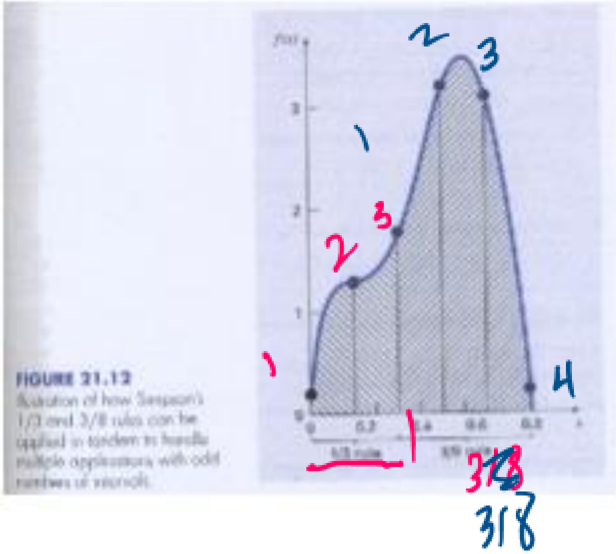
```
import math
def f(z):
    return (math.exp(-0.5*z**2)/((2.0*math.pi)**0.5))

n=10
a=-5.0
b=0.0
h=(b-a)/n
sum=f(a)+f(b)
for i in range(1,n):
   # print(i)
    k=a+i*h
    if i%2==0:
        #print("even number")
        sum=sum+2.0*f(k)
    else:
        #print("odd number")
        sum=sum+4.0*f(k)
sum=sum*h/3.0
print("The area is {} after {} iterations".format(sum,n))
```

Simpson's 3/8 Rule



- Simpson's 3/8 rule is Newton-Cotes with $n = 3$
- Chapra and Canale (2015)[5] illustrate Simpson's rule in the figure shown below Simpson's3/8 Rule

- Chapra and Canale (2015)[3] illustrate Simpson's rule in the figure shown below



Definition of Simpson's 3/8 Rule

- Chapra and Canale(2015)[6] provide the following definition of Simpson's rule

$$I = \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right]$$

where $h = (b-a)/3$

- Note that Simpson's 3/8 rule is of the form

$$I \approx (b-a)\frac{f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)}{8}$$

$$I \approx \text{width} \times \text{average height}$$

Truncation Error of Simpson's 3/8 Rule

$$E_t = -\frac{3}{80} h^5 f^{(4)}(\xi)$$

$$= -\frac{(b-a)^5}{6480} f^{(4)}(\xi)$$

Classroom Coding: One Interval of Simpson's 3/8 Rule

Notes

1. Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

2. Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

3. Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

4. https://www.codesansar.com/numerical-methods/integration-simpson-1-3-method-algorithm.htm

5. Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

6. Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*