MANE 3351

LECTURE 16

Classroom Management

Agenda

- Gaussian Quadrature
- Homework 4 (due 10/29/2025)
- Homework 5 (due November 3 - no late work)
- Lab 6 due before 9:30 AM
- Lab 7 after lecture (due 11/3/2025)

RESOURCES

Handouts

- Lecture 16 slides
- Lecture 16 slides marked
- Abscissas and Weights for Gaussian Integration

Calendar

Week	Monday Lecture	Wednesday Lecture
9	**10/27:** Gaussian Quadrature	**10/29:** Numerical Differentiation (not on Test 2)
10	**11/3:** Linear Algebra	**11/5:** Test 2 (Root Finding and Numerical Integration)

Lecture 16

Today's topic is Gaussian Quadrature $I = \sum_{i=1}^{n} w_i f(x_i)$

- $\int_{-1}^{1} f(x)dx \approx \sum_{i=1}^{n} w_i f(x_i) + R_n$
- x_i is the abscissa is the i-th zero of $P_n(x)$ (Legendre polynomial) Points
- w_i is the weight associate with x_i
- $R_n = \dfrac{2^{2n+1}(n!)^4}{(2n+1)[(2n)!]^3} f^{(2n)}(\xi)$
- Abscissas and weights are commonly tabled [1]

Python Code, part 1

```
# Part 1, original formula in range (-1,1)
#
import math
import numpy as np
def f(z):
    return (math.exp(-0.5*z**2)/((2.0*math.pi)**0.5))
#
# 2 point Gaussian Quadrature
x2=[-0.577350269189626,0.577350269189626]
w2=[1.0,1.0]
#
i2=0.0
for i in range(0,len(x2)):
    i2=i2+w2[i]*f(x2[i])
print("the area using 2-point gaussian quadrature is {}".format(i2))
#
# 5 point Gaussian Quadrature
x5=[0.0, -0.538469310105683,0.538469310105683,-0.906179845938664,0.906179845938664]
w5=[0.568888888888889,0.478628670499366,0.478628670499366,0.236926885056189,0.236926885056189]
i5=0.0
for i in range(0,len(x5)):
    i5=i5+w5[i]*f(x5[i])
print("the area using 5-point gaussain quadrature is {}".format(i5))
```

$\pm x_i$ $n=2$ w_i

0.57735 02691 89626	1.00000 00000 00000

$n=3$

0.00000 00000 00000	0.88888 88888 88889
0.77459 66692 41483	0.55555 55555 55556

for $n=2$

$x_1 = .57735 \cdots$ $\qquad w_1 = 1.0$

$x_2 = -.57735 \cdots$ $\qquad w_2 = 1.0$

$$I = \sum_{i=1}^{2} w_i f(x_i) = w_1 f(x_1) + w_2 f(x_2)$$

$$= 1.0 f(.57735\ldots) + 1.0 f(-.57735\ldots)$$

$\pm x_i$ $\qquad n = 2 \qquad w_i$

0.57735 02691 89626	1.00000 00000 00000

$n = 3$

0.00000 00000 00000	0.88888 88888 88889
0.77459 66692 41483	0.55555 55555 55556

for $n = 3$

$$I = \sum_{i=1}^{3} w_i f(x_i)$$

$x_1 = 0.0$

$x_2 = .774596669241483$

$x_3 = -.774596669241483$

$w_1 = 0.88889 \cdots$

$w_2 = 0.5555 \cdots$

$w_3 = 0.55555 \cdots$

Gauss' Formula, Arbitrary Interval

- $\int_a^b f\,(y)dy \approx \frac{b-a}{2}\sum_{i=1}^{n} w_i\, f(y_i) + R_n$
- $y_i = \left(\frac{b-a}{2}\right) x_i + \left(\frac{b+a}{2}\right)$
- $R_n = \frac{(b-a)^{2n+1}(n!)^4}{(2n+1)[(2n)!]^3} f^{(2n)}(\xi)$
- Note that x_i and w_i are the points and weights of Gaussian quadrature

Python Code, part 2

```
# Part 2
#
print("Gaussian Quadrature, arbitrary interval")
a=-5.0
b=0.0
#
# redo 2 point gaussian
i2=0.0
for i in range(0,len(x2)):
    y=((b-a)/2.0)*x2[i]+(b+a)/2.0
    i2=i2+w2[i]*f(y)
i2=((b-a)/2.0)*i2
print("the area using 2-point gaussian quadrature is {}".format(i2))
#
# redoc 5 point gaussian
i5=0.0
for i in range(0,len(x5)):
    y=((b-a)/2.0)*x5[i]+(b+a)/2.0
    i5=i5+w5[i]*f(y)
i5=((b-a)/2.0)*i5
print("the area using 5-point gaussian quadrature is {}".format(i5))
```

$\pm x_i$		w_i
	$n=2$	
0.57735 02691 89626		1.00000 00000 00000
	$n=3$	
0.00000 00000 00000		0.88888 88888 88889
0.77459 66692 41483		0.55555 55555 55556

$$y_i = \frac{(b-a)}{2} x_i + \frac{(b+a)}{2}$$

$$I = \frac{(b-a)}{2} \sum_{i=1}^{n} w_i f(y_i)$$

$a=-5 \quad b=0$

$$x_1 = .57735 \to y_1 = \left(\frac{0 - -5}{2}\right) .57735\ldots + \frac{(0 + -5)}{2} = -1.05663\ldots$$

$$x_2 = -.57735\ldots \to y_2 = \left(\frac{0 - -5}{2}\right)(-.57735) + \left(\frac{0+-5}{2}\right)$$

$$= -3.94338$$

$$I = \left(\frac{0 - -5}{2}\right)\left[w_1 f(y_1) + w_2 f(y_2)\right]$$

$$= 2.5\left[1.0 f(-1.05663) + 1.0 f(-3.94338)\right.$$

ChatGPT program

- ChatGPT can be a useful tool for developing Python code.
- It should not replace your knowledge and expertise
- ChatGPT can be wrong

Notes

1. Abramowitz, M., and Stegun, I. (1972), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*