MANE 3351

LECTURE 20

Classroom Management

Agenda

– Summary of operations on Vectors, Matrix Multiplication, Three-Dimensional Transformations

– Determinants

– Introduction to Inverse Matrix

– Lab Assignment 8 (assigned 11/11/25, due 11/18/25 before 9:30 AM)

RESOURCES

Handouts

- Lecture 20 slides
- Lecture 20 slides marked]
- Linear Algebra Handouts

Calendar

Week	Monday Lecture	Wednesday Lecture
11	**11/10:** Test 2	**11/12:** Lecture 20
12	**11/17:** Lecture 21	**11/19:** Lecture 22
13	**11/24:** Lecture 23	**11/26:** Lecture 24
14	**12/1:** Lecture 25	**12/3:** Lecture 26
15	**12/8:** Lecture 27	**12/10:** Review

Final Exam is Monday 12/15/2025 8:00 - 9:45 AM

I will be off-campus participating in an ABET visit and a proctor will be arranged for the final exam.

Lecture 20 Overview

– Vector Operations,
– Determinants, and
– Matrix Inversion

Vectors

A **vector** $\mathbf{x} \in \mathbb{R}^n$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

can be thought of as a one-dimensional array of numbers and is written as

- $\mathbf{x}$ is often called a column vector
- the dimension of $\mathbf{x}$ is $(n \times 1)$

A row vector can be written as

$$\mathbf{y} = [y_1 \ y_2 \ \cdots \ y_m]$$

– the dimension of $\mathbf{y}$ is $(1 \times m)$

Scalar Product

$$\alpha\mathbf{x} = \begin{bmatrix} \alpha x_1 \\ \alpha_2 x_2 \\ \vdots \\ \alpha x_n \end{bmatrix}$$

for α (a constant or scalar)

Addition/Subtraction

$$\mathbf{x} \pm \mathbf{y} = \begin{bmatrix} x_1 \pm y_1 \\ x_2 \pm y_2 \\ \vdots \\ x_n \pm y_n \end{bmatrix}$$

– Note that the dimensions of $\mathbf{x}$ and $\mathbf{y}$ must be identical

Matrices

A **matrix** is a two-dimensional array of numbers written as

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}$$

- A matrix has two dimensions and can be written as $A_{n \times m}$ where n is the number of rows and m is the number of columns
- A column vector can be considered an $n \times 1$ matrix and a row vector can be considered an $1 \times m$ matrix
- Matrices may or may not be square

Determinant

- The Determinant Video
- Determinant of a Matrix Website

Ex: Determinant of a 2x2 Full Rank Matrix

EX: Determinant of a 2x2 Non-Full Rank Matrix

Ex: Shortcut for Determinant of a 3x3 Matrix

For a 3×3 Matrix

For a 3×3 matrix (3 rows and 3 columns):

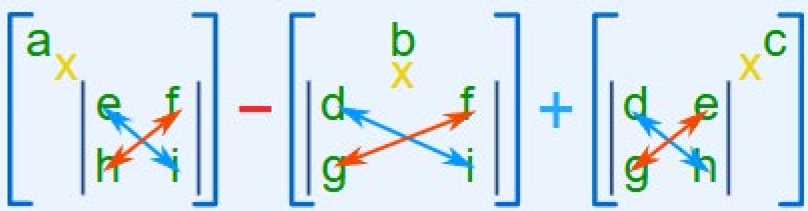
$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

The determinant is:

$$|A| = a(ei - fh) - b(di - fg) + c(dh - eg)$$

"The determinant of A equals ... etc"

It may look complicated, but **there is a pattern**:



Ex: Method of Minors for finding determinant of 3x3 or higher dimension Matrix

Inverse Matrix

Inverse Matrices Video

Linear Algebra Handout