

MANE 3351

LECTURE 20

Classroom Management

Agenda

- Summary of operations on Vectors, Matrix Multiplication, Three-Dimensional Transformations
- Determinants
- Introduction to Inverse Matrix
- Lab Assignment 8 (assigned 11/11/25, due 11/18/25 before 9:30 AM)

RESOURCES

Handouts

- [Lecture 20 slides](#)
- Lecture 20 slides marked]
- [Linear Algebra Handouts](#)

Calendar

Week	Monday Lecture	Wednesday Lecture
11	11/10: Test 2	11/12: Lecture 20
12	11/17: Lecture 21	11/19: Lecture 22
13	11/24: Lecture 23	11/26: Lecture 24
14	12/1: Lecture 25	12/3: Lecture 26
15	12/8: Lecture 27	12/10: Review

**Final Exam is Monday 12/15/2025 8:00 - 9:45
AM**

I will be off-campus participating in an ABET visit
and a proctor will be arranged for the final
exam.

Lecture 20 Overview

- Vector Operations,
- Determinants, and
- Matrix Inversion

Vectors

A **vector** $\mathbf{x} \in \mathbb{R}^n$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

can be thought of as a one-dimensional array of numbers and is written as

- \mathbf{x} is often called a column vector
- the dimension of \mathbf{x} is $(n \times 1)$

A row vector can be written as

$$\mathbf{y} = [y_1 \ y_2 \ \cdots \ y_m]$$

– the dimension of \mathbf{y} is $(1 \times m)$

Scalar Product

$$\alpha \mathbf{X} = \begin{bmatrix} \alpha x_1 \\ \alpha x_2 \\ \vdots \\ \alpha x_n \end{bmatrix}$$

for α (a constant or scalar)

Addition/Subtraction

$$\mathbf{x} \pm \mathbf{y} = \begin{bmatrix} x_1 \pm y_1 \\ x_2 \pm y_2 \\ \vdots \\ x_n \pm y_n \end{bmatrix}$$

- Note that the dimensions of \mathbf{x} and \mathbf{y} must be identical

Matrices

A **matrix** is a two-dimensional array of numbers written as

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}$$

- A matrix has two dimensions and can be written as $A_{n \times m}$ where n is the number of rows and m is the number of columns
- A column vector can be considered an $n \times 1$ matrix and a row vector can be considered an $1 \times m$ matrix
- Matrices may or may not be square

Determinant

- The Determinant Video
- Determinant of a Matrix Website

Ex: Determinant of a 2x2 Full Rank Matrix

EX: Determinant of a 2x2 Non-Full Rank Matrix

Ex: Shortcut for Determinant of a 3x3 Matrix

For a 3×3 Matrix

For a 3×3 matrix (3 rows and 3 columns):

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

The determinant is:

$$|A| = a(ei - fh) - b(di - fg) + c(dh - eg)$$

"The determinant of A equals ... etc"

It may look complicated, but **there is a pattern**:

The diagram shows the calculation of the determinant of a 3x3 matrix using the rule of Sarrus. It consists of three terms in brackets, separated by minus and plus signs. Each term is a 2x2 determinant. The first term is $a \begin{vmatrix} e & f \\ h & i \end{vmatrix}$, the second is $-b \begin{vmatrix} d & f \\ g & i \end{vmatrix}$, and the third is $+c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$. The elements a, b, c are in yellow, and the elements $e, f, h, i, d, f, g, i, d, e, g, h$ are in green. Blue arrows show the positive paths (down-right) and red arrows show the negative paths (down-left) for each 2x2 determinant.

$$\left[a \begin{vmatrix} e & f \\ h & i \end{vmatrix} \right] - \left[b \begin{vmatrix} d & f \\ g & i \end{vmatrix} \right] + \left[c \begin{vmatrix} d & e \\ g & h \end{vmatrix} \right]$$

**Ex: Method of Minors for finding determinant
of 3x3 or higher dimension Matrix**

Inverse Matrix

Inverse Matrices Video

Linear Algebra Handout