

MANE 3351

Test 2

```
describe(TestScores$`Test 2`)
```

```
##      vars  n  mean    sd median trimmed  mad min max range  skew kurtosis   se  
## X1      1 21 82.48 18.74     91   85.94 13.34  35 100    65 -1.35     0.98 4.09
```

```
stem(TestScores$`Test 2`,scale=2)
```

```
##
```

```
## The decimal point is 1 digit(s) to the right of the |
```

```
##
```

```
## 3 | 56
```

```
## 4 |
```

```
## 5 |
```

```
## 6 | 5
```

```
## 7 | 0359
```

```
## 8 | 035
```

```
## 9 | 12345556
```

```
## 10 | 000
```

```
boxplot(TestScores$`Test 2`)
```

```
summary(TestScores)
```

```
##      Username      Test 1      Test 2      Diff
## Length:21      Min.   : 20.00      Min.   : 35.00      Min.   : -39.000
## Class :character 1st Qu.: 75.00      1st Qu.: 75.00      1st Qu.: -5.000
## Mode  :character Median : 87.00      Median : 91.00      Median :  4.000
##                Mean   : 79.57      Mean   : 82.48      Mean   :  2.905
##                3rd Qu.: 90.00      3rd Qu.: 95.00      3rd Qu.:  9.000
##                Max.   :100.00      Max.   :100.00      Max.   : 38.000
##      Average
## Min.   : 27.50
## 1st Qu.: 76.50
## Median : 87.00
## Mean   : 81.02
## 3rd Qu.: 92.00
## Max.   :100.00
```

Lecture 21

Classroom Management

Agenda

- Inverting Matrices by Hand
- Transpose of a Matrix
- Python for Linear Algebra
- Homework 6
- Lab 9 for those who did not finish on Monday

Resources

Handouts

- [Lecture 21 slides](#)
- Lecture 21 slides marked
- [Python Linear Algebra Cheat Sheet](#)

Calendar

Week	Monday Lecture	Wednesday Lecture
12	11/17: Lecture 21	11/19: Lecture 22
13	11/24: Lecture 23	11/26: Lecture 24
14	12/1: Lecture 25	12/3: Lecture 26
15	12/8: Lecture 27	12/10: Review

Final Exam is Monday 12/15/2025 8:00 - 9:45 AM

I will be off-campus participating in an ABET visit and a proctor will be arranged for the final exam.

Assignments

- Homework 6 (assigned 11/17, due 11/24)
- Lab Assignment 8 (assigned 11/12/25, due 11/19/25 before 9:30 AM)

Inverting 2 x 2 Matrices by Hand

- Inverting 2x2

$$Ax = b$$

Solve for x

$$A^{-1} A x = A^{-1} b$$

$$I x = A^{-1} b$$

$$x = A^{-1} b$$

Simple Alg.

$$2x = 6$$

$$x = \frac{6}{2} = 3$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\det(A) = |A| = ad - bc$$

for inverse

matrix to exist $|A| \neq 0$

$$5 \times 3 - (-1 \times 2) = 17$$

$$15 \text{ (crossed out)}$$

$$A = \begin{pmatrix} 5 & -1 \\ 2 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{17} \begin{pmatrix} 3 & 1 \\ -2 & 5 \end{pmatrix}$$

Ex: Inverting a 2x2 Matrix by Hand

Inverting 3x3 Matrices by Hand

- [Inverting 3x3](#)

Ex: Inverting a 3x3 Matrix by Hand

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

Matrix of minors

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

$$1 \cdot 0 - 6 \cdot 4 = -24 \quad 0 \cdot 0 - 5 \cdot 4 = -20 \quad 0 \cdot 6 - 5 \cdot 1 = -5$$

$$2 \cdot 0 - 6 \cdot 3 = -18 \quad 1 \cdot 0 - 5 \cdot 3 = -15 \quad 1 \cdot 6 - 5 \cdot 2 = -4$$

$$2(4) - 1(3) = 5 \quad 1(4) - 0(3) = 4 \quad (1)1 - 0(2) = 1$$

$$\begin{pmatrix} -24 & -20 & -5 \\ -18 & -15 & -4 \\ 5 & 4 & 1 \end{pmatrix} \rightarrow$$

$$\det = 1(-24) - (2 \times -20) + 3(-5)$$

Checker board $\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$

$$\begin{pmatrix} -24 & 20 & -5 \\ 18 & -15 & 4 \\ 5 & -4 & 1 \end{pmatrix}$$

Matrix of cofactors

$$\begin{pmatrix} -24 & 20 & -5 \\ 18 & -15 & 4 \\ 5 & -4 & 1 \end{pmatrix} \xrightarrow{\text{Transpose}} \begin{pmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & -4 & 1 \end{pmatrix}$$

Red dashed arrows indicate the transpose operation: (1,2) to (2,1), (1,3) to (3,1), and (2,3) to (3,2).

Since $\det = 1$

The inverse is

$$\frac{1}{1} \begin{pmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & -4 & 1 \end{pmatrix}$$

ChatGPT Solution

$$A^{-1} = \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix}$$

Transpose of a Matrix

- [Matrix Transpose](#)

Python Linear Algebra Cheat Sheet

- Jupyter Notebook Demonstration

```
import numpy as np
from scipy import linalg, sparse
# Creating matrices

A=np.mat([[3,4],[5,6]])
print(A)

print(A.I)
print(linalg.inv(A))

print( A.I @ A)

C=np.mat([[1,2,5],[5,8,2],[7,6,5]])
print(C)

rhs = ([4],[25],[-3])
print(rhs)

print(C.I @ rhs)
print(linalg.solve(C,rhs))

print(A.T)

print(linalg.det(C))

D= np.matrix(np.random.random((2,2)))
print(D)
```