MANE 3351

Lecture 22

Classroom Management

Agenda

- Row Echelon form
- Lab 9 (assigned 11/19/2025, due 11/26/2025)

Resources

Handouts

- Lecture 22 slides
- Lecture 22 slides marked

Calendar

Week	Monday Lecture	Wednesday Lecture
12	**11/17:** Lecture 21	**11/19:** Lecture 22
13	**11/24:** Lecture 23	~~**11/26:** Lecture 24~~
14	**12/1:** Lecture 25	**12/3:** Lecture 26
15	**12/8:** Lecture 27	**12/10:** Review

no class

Final Exam is Monday 12/15/2025 8:00 - 9:45 AM

I will be off-campus participating in an ABET visit and a proctor will be arranged for the final exam.

Assignments

- Homework 6 (assigned 11/17, due 11/24)
- Lab Assignment 9 (assigned 11/19/25, due 11/26/25 before 9:30 AM)

Row Echelon Form

$(A \mid b)$

$Ax = b$

$A^{-1}Ax = A^{-1}b$

$Ix = A^{-1}b$

$x = A^{-1}b$

- Augmented Matrix
- Row operations
  - Any two rows can be interchanged (next week)
  - The elements of any row can be multiplied by a nonzero real number
  - Any row can be changed by adding or subtracting the corresponding elements with another row.

Row Echelon Form Video

$$2x - 3y = 18$$
$$5x + 2y = 7$$

$Az = b$

$A = \begin{pmatrix} 2 & -3 \\ 5 & 2 \end{pmatrix}$, $b = \begin{pmatrix} 18 \\ 7 \end{pmatrix}$

$A_{(2\times2)}$

$z = \begin{pmatrix} x \\ y \end{pmatrix}$

$z_{(2\times1)}$

Augmented matrix

$\left(\begin{array}{cc|c} 2 & -3 & 18 \\ 5 & 2 & 7 \end{array}\right)$

$-5R_1$, $2R_2$

$$\begin{bmatrix} 2 & -3 & 18 \\ 5 & 2 & 7 \end{bmatrix}$$

$$\begin{bmatrix} -10 & 15 & -90 \\ 10 & 4 & 14 \end{bmatrix}$$

$R_2 + R_1$

$$\begin{bmatrix} -10 & 15 & -90 \\ 0 & 19 & -76 \end{bmatrix}$$

$-\frac{1}{10}R_1$, $\frac{1}{19}R_2$

$$\begin{bmatrix} 1 & -\frac{3}{2} & 9 \\ 0 & 1 & -4 \end{bmatrix}$$

1. always add instructions (red text)

2. steps are not unique

3. $$\begin{bmatrix} -10 & 15 & -90 \\ 0 & 19 & -76 \end{bmatrix}$$

$19y = -76$

$-10x + 15(-4) = -90$

$$\begin{matrix} -5R_1 \\ 2R_2 \end{matrix}\left[\begin{array}{cc|c} 2 & -3 & 18 \\ 5 & 2 & 7 \end{array}\right]$$

$$\begin{pmatrix} 2 & -3 & 18 \\ 5 & 2 & 7 \end{pmatrix}$$

$$\begin{matrix} \\ R_2+R_1 \end{matrix}\left[\begin{array}{cc|c} -10 & 15 & -90 \\ 10 & 4 & 14 \end{array}\right] \quad 5R_1-2R_2$$

$$\begin{matrix} -\frac{1}{10}R_1 \\ \frac{1}{19}R_2 \end{matrix}\left[\begin{array}{cc|c} -10 & 15 & -90 \\ 0 & 19 & -76 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -\frac{3}{2} & 9 \\ 0 & 1 & -4 \end{array}\right]$$

$$\begin{pmatrix} 2 & -3 & 18 \\ 5(2)-2(5) & 5(-3)-2(2) & 5(18)-2(76) \end{pmatrix}$$

$$\begin{matrix} \frac{1}{2}R_1 \\ \frac{1}{-19}R_2 \end{matrix}\begin{pmatrix} 2 & -3 & 18 \\ 0 & -19 & 76 \end{pmatrix}$$

Example: 2 equations and 2 unknowns

$$\begin{cases} 2x + 3y = 7 \\ 4x - y = 5 \end{cases}$$

$Ax = b$

$$A = \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix} \quad x = \begin{pmatrix} x \\ y \end{pmatrix} \quad b = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$$

$R_2 - 2R_1$

$$\begin{pmatrix} 2 & 3 & 7 \\ 4 & -1 & 5 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & 3 & 7 \\ 0 & -7 & -9 \end{pmatrix}$$

$\frac{1}{2} R_1$

$-\frac{1}{7} R_2$

$$\begin{pmatrix} 2 & 3 & 7 \\ 0 & -7 & -9 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 3/2 & 7/2 \\ 0 & 1 & 9/7 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3/2 & 7/2 \\ 0 & 1 & \frac{9}{7} \end{pmatrix}$$

$$y = \frac{9}{7}$$

$$x + \frac{3}{2}y = \frac{7}{2}$$

$$x + \frac{3}{2}\left(\frac{9}{7}\right) = \frac{7}{2}$$

$$x + \frac{27}{14} = \frac{7}{2}$$

$$x = \frac{49}{14} - \frac{27}{14} = \frac{22}{14} = \frac{11}{7}$$

$$\begin{cases} x+y+z=6 \\ 2x-y+3z=14 \\ 3x+4y-2z=2 \end{cases}$$

$R_2 - 2R_1$

$R_3 - 3R_1$

$$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 14 \\ 3 & 4 & -2 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 2 \\ 0 & 1 & -5 & -16 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 2 \\ 0 & 1 & -5 & -16 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 2 \\ 0 & 0 & -14 & -46 \end{pmatrix}$$

$R_3 + \frac{1}{3}R_2$

$3R_3 + R_2$

$$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 2 \\ 0 & 0 & -14 & -46 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & -\frac{1}{3} & -\frac{2}{3} \\ 0 & 0 & 1 & \frac{46}{14} \end{pmatrix}$$

$-\frac{1}{3}R_2$

$-\frac{1}{14}R_3$

$$1z = \frac{46}{14} = \frac{23}{7}$$

$$y - \frac{1}{3}z = -2/3$$

$$y - \frac{1}{3}\left(\frac{23}{7}\right) = -2/3$$

$$y + -\frac{23}{21} = -2/3$$

$$y = \frac{-14}{21} + \frac{23}{21}$$

$$y = \frac{9}{21} = \frac{3}{7}$$

$$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & -\frac{1}{3} & -\frac{2}{3} \\ 0 & 0 & 1 & \frac{46}{14} \end{pmatrix}$$

$z = \frac{23}{7}$

$y = 3/7$

$x + y + 3 = 6$

$x + \frac{3}{7} + \frac{23}{7} = 6$

$x + \frac{26}{7} = \frac{42}{7}$

$x = \frac{42-26}{7} = \frac{16}{7}$

Example: 3 equations and 3 unknowns