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Wednesday, October 23, 2024 10:51 AM

Section 1

MANE 3351

Subsection 1

Lecture 16

Classroom Management

Agenda

- Gaussian Quadrature
- Homework 5 (due October - no late work)
- Lab session postponed

Subsection 2

Resources

Handouts

- Lecture 16 slides
- Lecture 16 slides marked
- Abscissas and Weights for Gaussian Integration

Calendar

Lecture/Lab	Date	Topic
10/21	No Class, Dr. Timmer on ABET Visit	No Class, Dr. Timmer on ABET Visit
10/23	Gaussian Quadrature, Homework 5	Lab 8 postponed
10/28	Numerical Differentiation (not on Test 2)	Lab 8
11/4	Numerical Integration (not on Test 2)	Lab 8, continued
11/6	Linear Algebra, part 1 (not on Test 2)	Lab 9
11/11	**Test 2 (Root Finding and Numerical Integration)**	No Lab

Lecture 16

Points

Today's topic is Gaussian Quadrature

- $\int_{-1}^{1} f(x)dx \approx \sum_{i=1}^{n} w_i f(x_i) + R_n$
- x_i is the abscissa is the i-th zero of $P_n(x)$ (Legendre polynomial)
- w_i is the weight associate with x_i
- $R_n = \frac{2^{2n+1}(n!)^4}{(2n+1)[(2n)!]^3} f^{(2n)}(\xi)$
- Abscissas and weights are commonly tabled [1]

[1] Abramowitz, M., and Stegun, I. (1972), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*

Python Code, part 1

```
# Part 1, original formula in range (-1,1)
#
import math
import numpy as np
def f(z):
    return (math.exp(-0.5*z**2)/((2.0*math.pi)**0.5))
#
# 2 point Gaussian Quadrature
x2=[-0.577350269189626,0.577350269189626]
w2=[1.0,1.0]
#
i2=0.0
for i in range(0,len(x2)):
    i2=i2+w2[i]*f(x2[i])
print("the area using 2-point gaussian quadrature is {}".forma
#
# 5 point Gaussian Quadrature
```

Gauss' Formula, Arbitrary Interval

- $\int_a^b f(y)dy \approx \frac{b-a}{2} \sum_{i=1}^{n} w_i f(y_i) + R_n$
- $y_i = \left(\frac{b-a}{2}\right) x_i + \left(\frac{b+a}{2}\right)$
- $R_n = \frac{(b-a)^{2n+1}(n!)^4}{(2n+1)[(2n)!]^3} f^{(2n)}(\xi)$
- Note that x_i and w_i are the points and weights of Gaussian quadrature

Python Code, part 2

```
# Part 2
#
print("Gaussian Quadrature, arbitrary interval")
a=-5.0
b=0.0
#
# redo 2 point gaussian
i2=0.0
for i in range(0,len(x2)):
    y=((b-a)/2.0)*x2[i]+(b+a)/2.0
    i2=i2+w2[i]*f(y)
i2=((b-a)/2.0)*i2
print("the area using 2-point gaussian quadrature is {}".forma
#
# redoc 5 point gaussian
i5=0.0
for i in range(0,len(x5)):
```

finding y_i

ChatGPT program

- ChatGPT can be a useful tool for developing Python code.
- It should not replace your knowledge and expertise
- ChatGPT can be wrong

	Lecture/Lab	Date	Topic
M	10/21	No Class, Dr. Timmer on ABET Visit	No Class, Dr. Timmer on ABET Visit
W	10/23	Gaussian Quadrature, Homework 5	Lab 8 postponed
M	10/28	Numerical Differentiation (not on Test 2)	Lab 8
W	~~11/4~~ 10/30	Numerical Integration (not on Test 2)	Lab 8, continued
M	~~11/6~~ 11/4	Linear Algebra, part 1 (not on Test 2)	Lab 9
W	~~11/11~~ 11/6	**Test 2 (Root Finding and Numerical Integration)**	No Lab

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NUMERICAL ANALYSIS

Table 25.4 ABSCISSAS AND WEIGHT FACTORS FOR GAUSSIAN INTEGRATION

$$\int_{-1}^{+1} f(x)dx \approx \sum_{i=1}^{n} w_i f(x_i)$$

Abscissas $= \pm x_i$ (Zeros of Legendre Polynomials) Weight Factors $= w_i$

$\pm x_i$	w_i	$\pm x_i$	w_i
$n=2$		$n=8$	
0.57735 02691 89626	1.00000 00000 00000	0.18343 46424 95650	0.36268 37833 78362
		0.52553 24099 16329	0.31370 66458 77887
		0.79666 64774 13627	0.22238 10344 53374
		[illegible]	0.10122 85362 90376

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$\pm\, y_i$ w_i

$n = 5$

→ 0.00000 00000 00000	0.56888 88888 88889
0.53846 93101 05683	0.47862 86704 99366
0.90617 98459 38664	0.23692 68850 56189

$-.53846\ldots$ $.47862\cdots$

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$-.90617\cdots$ $.23692\cdots$

$I = \int_{-5}^{0} \phi(z)\,dz \leftarrow$ pdf of Standard normal

$\cancel{A}\ f(x) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2} z^2\right]$

$a = -5,\ b = 0$

$n = 3$

$x_1 = 0.\cancel{9}77459$

$x_2 = .77459$

$x_3 - .77459$

$w_1 = .88889$

$w_2 = .55556$

$w_3 = .5556$

$y_1 = \left(\frac{b-a}{2}\right)x_1 + \left(\frac{b+a}{2}\right) = \left(\frac{0--5}{2}\right)0.0 + \left(\frac{-5+0}{2}\right) = -2.5$

$y_2 = \left(\frac{b-a}{2}\right)x_2 + \left(\frac{b+a}{2}\right) = \left(\frac{0--5}{2}\right)(.77459) + \left(\frac{-5+0}{2}\right) = -0.5635$

$y_3 = \left(\frac{b-a}{2}\right)x_3 + \frac{(b+a)}{2} = \left(\frac{0--5}{2}\right)(-.7749) + \left(\frac{-5+0}{2}\right) = -4.4365$

$I = \left(\frac{b-a}{2}\right)\sum_{i=1}^{3} w_i f(y_i)$

$= \left(\frac{0--5}{2}\right)\left[w_1 f(y_1) + w_2 f(y_2) + w_3 f(y_3)\right]$