# Printout

Monday, November 11, 2024 12:50 PM

# Section 1

## MANE 3351

## Subsection 1

### Lecture 20

# Classroom Management

## Agenda

- Summary of operations on Vectors, Matrix Multiplication, Three-Dimensional Transformations
- Determinants
- Introduction to Inverse Matrix
- Lab Assignment 9 today

Subsection 2

## Resources

## Handouts

- Lecture 20 slides
- Lecture 20 slides marked
- Linear Algebra Handout

# Calendar

| Date | Lecture Topic | Lab Topic |
|---|---|---|
| 11/11 | Lecture 20 - Determinant | Lab 9 |
| 11/13 | | |
| 11/18 | | |
| 11/20 | | |
| 11/25 | | |
| 11/27 | | |
| 12/2 | | |
| 12/4 | | |
| 12/9 | **Final exam 1:15 - 3:00 pm** | |

## Lecture 20 Overview

- Vector Operations,
- Determinants, and
- Matrix Inversion

# Vectors

A **vector** $\mathbf{x} \in \mathbb{R}^n$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

can be thought of as a one-dimensional array of numbers and is written as

- $\mathbf{x}$ is often called a column vector
- the dimension of $\mathbf{x}$ is $(n \times 1)$

A row vector can be written as

$$\mathbf{y} = \begin{bmatrix} y_1 \; y_2 & \cdots & y_m \end{bmatrix}$$

- the dimension of $\mathbf{y}$ is $(1 \times m)$

## Scalar Product

$$\alpha \mathbf{x} = \begin{bmatrix} \alpha x_1 \\ \alpha_2 x_2 \\ \vdots \\ \alpha x_n \end{bmatrix}$$

for $\alpha$ (a constant or scalar)

## Addition/Subtraction

$$\mathbf{x} \pm \mathbf{y} = \begin{bmatrix} x_1 \pm y_1 \\ x_2 \pm y_2 \\ \vdots \\ x_n \pm y_n \end{bmatrix}$$

- Note that the dimensions of $\mathbf{x}$ and $\mathbf{y}$ must be identical

## Matrices

A **matrix** is a two-dimensional array of numbers written as

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}$$

- A matrix has two dimensions and can be written as $A_{n \times m}$ where $n$ is the number of rows and $m$ is the number of columns
- A column vector can be considered an $n \times 1$ matrix and a row vector can be considered an $1 \times m$ matrix
- Matrices may or may not be square

Operations

$$\alpha A = \begin{bmatrix} \alpha a_{11} & \alpha a_{21} & \cdots & \alpha a_{1m} \\ \vdots & \vdots & \ddots & \vdots \end{bmatrix}$$



Addition / Subtraction $\rightarrow$ same dimension

Multiplication $A_{(r_1 \times c_1)}$ $B_{(r_2 \times c_2)}$ $\rightarrow$ $c_1 = r_2$ to perform multiplication

## Determinant

- The Determinant Video
- Determinant of a Matrix Website

## Ex: Determinant of a 2x2 Full Rank Matrix

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

off diagonal

main diagonal

$$|A| = 1 \cdot 4 - 2 \cdot 3 = -2$$

## EX: Determinant of a 2x2 Non-Full Rank Matrix

$$\cancel{A} \; B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \quad |B| = 1 \cdot 4 - 2 \cdot 2 = \emptyset$$

## Ex: Shortcut for Determinant of a 3x3 Matrix

$$C = \begin{bmatrix} 1 & 4 & 9 \\ 2 & 5 & 8 \\ 3 & 6 & 7 \end{bmatrix} \rightarrow \text{augmented Matrix}$$

$$\left[\begin{array}{ccc|cc} 1 & 4 & 9 & 1 & 4 \\ 2 & 5 & 8 & 2 & 5 \\ 3 & 6 & 7 & 3 & 9 \end{array}\right]$$

$$\det(C) = |C| = \{1\cdot5\cdot7 + 4\cdot8\cdot3 + 9\cdot2\cdot6\} - \{3\cdot5\cdot9 + 6\cdot8\cdot1 + 7\cdot2\cdot4\}$$

$$= 35 + 96 + 108 - \{135 + 48 + 56\}$$

$$= 239 - 239$$

$$= \emptyset$$

$$+ \quad \begin{vmatrix} 1 & 4 & 9 \\ 2 & 5 & 8 \\ 3 & 6 & 7 \end{vmatrix} \qquad 1 \cdot \begin{vmatrix} 5 & 8 \\ 6 & 7 \end{vmatrix} = 1(35-48) = -13$$

$$- \quad \begin{vmatrix} 1 & 4 & 9 \\ 2 & 5 & 8 \\ 3 & 6 & 7 \end{vmatrix} \qquad -4 \begin{vmatrix} 2 & 8 \\ 3 & 7 \end{vmatrix} = -4 \times (14-24) = +40$$



$$+ \quad \begin{matrix} 1 & 4 & 9 \\ 2 & 5 & 8 \\ 3 & 6 & 7 \end{matrix} \qquad 9 \begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} = 9(12-15) = -27$$

$$|k| = -13 + 40 - 27 = \emptyset$$

Ex: Method of Minors for finding determinant of 3x3 or higher dimension Matrix

$2x = 6 \rightarrow \frac{2}{2}x = \frac{6}{2} \rightarrow x = 3$

division

Identity Matrix
square $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$



# Inverse Matrix

A matrix
x & b are vectors

$2 \cdot 1 = 2$

$IA = A$

$Ax = b$

$\underbrace{A^{-1}A}x = A^{-1}b$

$Ix = A^{-1}b$

$x = A^{-1}b$

Inverse Matrices Video

# Linear Algebra Handout