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Monday, October 14, 2024 11:55 AM

Section 1

MANE 3351

Lecture 14

Classroom Management

Agenda

- Return Test 1
- Simpson's Rule
- Lab 7
- Homework 3 due today
- Homework 4

Calendar

Date	Lecture	Lab
10/14	Simpson's Rule, Homework 4	Lab 7
10/16	Romberg Integration	Lab 7, continued
10/21	No Class, Dr. Timmer on ABET Visit	No Class, Dr. Timmer on ABET Visit
10/23	Gaussian Quadrature, Homework 5	Lab 8
10/28	Numerical Differentiation (not on Test 2)	Lab 8, continued
11/4	Numerical Integration (not on Test 2)	Lab 9
11/6	Linear Algebra, part 1 (not on Test 2)	Lab 9, continued
11/11	**Test 2 (Root Finding**	No Lab

Resources

Handouts

- Lecture 14 Slides
- Lecture 14 Marked Slides

Lecture 14

Today's topic is Simpson's Rule.

- Simpson's (1/3) rule is Newton-Cotes with $n = 2$
- Chapra and Canale (2015)[a] illustrate Simpson's rule in the figure shown below Simpson's Rule

[a] Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

Definition of Simpson's Rule

- Chapra and Canale(2015)[a] provide the following definition of Simpson's rule

$$I = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$$

where $h = (b-a)/2$

- Note that Simpson's rule is of the form

$$I \approx (b-a)\,\frac{f(x_0) + 4f(x_1) + f(x_2)}{6}$$
$$I \approx \text{ width } \times \text{ average height}$$

[a] Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

Simpson's Rule Error Analysis

- Chapra and Canale (2015)[a] provides the following definition:

$$I = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right] - \frac{1}{90}f^{(4)}(\xi)h^5$$

$I =$ Simpson's 1/3 approximation $-$ Truncation error

error term

[a] Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

lower end point
odd interior points
even interior points
upper endpoints

Multiple Applications of Simpson's Rule

- The number of segments must be even
- The formula is

$$I \approx (b-a)\,\frac{f(x_0) + 4\sum_{i=1,3,5}^{n-1} f(x_i) + 2\sum_{j=2,4,6}^{n-2} f(x_j) + f(x_n)}{3n}$$

where $h = \frac{b-a}{n}$

Pseudo-code: Simpson's 1/3 Rule

- CodeSansar[a] provides the following pseudo-code Simpson's 1/3 pseudo-code

[a] https://www.codesansar.com/numerical-methods/integration-simpson-1-3-method-algorithm.htm

Python Code for Multiple Simpson's 1/3 Rule

```
import math
def f(z):
    return (math.exp(-0.5*z**2)/((2.0*math.pi)**0.5))

n=10
a=-5.0
b=0.0
h=(b-a)/n
sum=f(a)+f(b)
for i in range(1,n):
    # print(i)
    k=a+i*h
    if i%2==0:
        #print("even number")
        sum=sum+2.0*f(k)
    else:
        #print("odd number")
```

Simpson's 3/8 Rule

- Simpson's 3/8 rule is Newton-Cotes with $n = 3$
- Chapra and Canale (2015)[a] illustrate Simpson's rule in the figure shown below Simpson's3/8 Rule

[a] Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

Definition of Simpson's 3/8 Rule

- Chapra and Canale(2015)[a] provide the following definition of Simpson's rule

$$I = \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right]$$

where $h = (b - a)/3$

- Note that Simpson's 3/8 rule is of the form

$$I \approx (b - a)\,\frac{f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)}{8}$$

$$I \approx \text{ width} \times \text{ average height}$$

[a] Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

Truncation Error of Simpson's 3/8 Rule

$$E_t = -\frac{3}{80} h^5 f^{(4)}(\xi)$$
$$= -\frac{(b-a)^5}{6480} f^{(4)}(\xi)$$

Classroom Coding: One Interval of Simpson's 3/8 Rule

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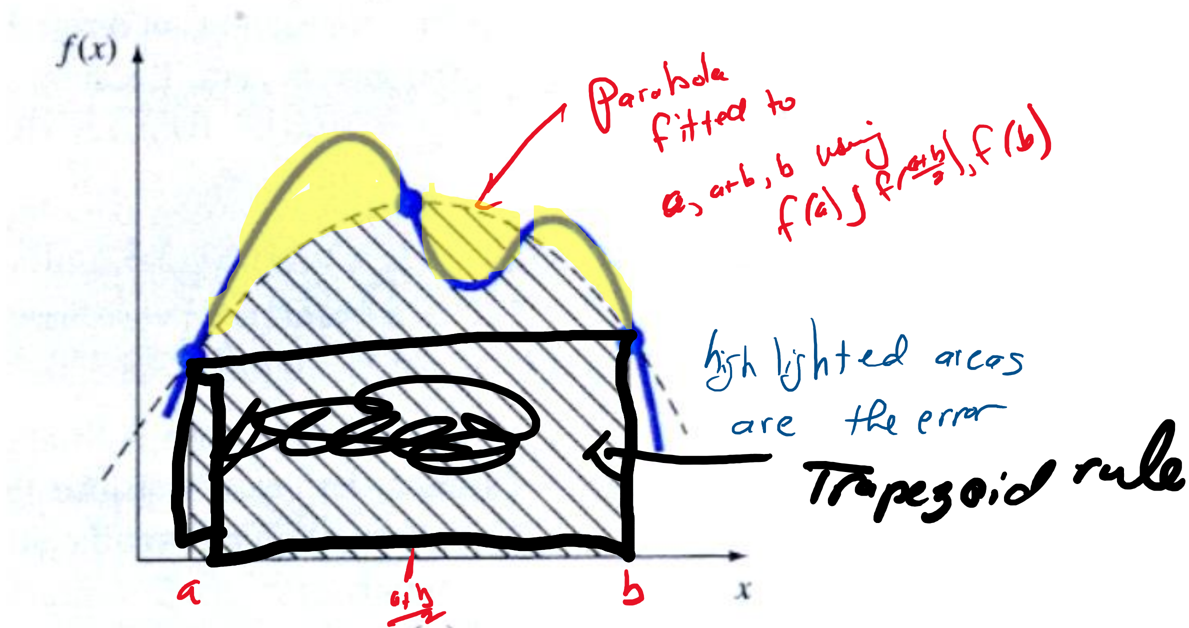
Lecture 14

Today's topic is Simpson's Rule.

- Simpson's (1/3) rule is Newton-Cotes with $n = 2$
- Chapra and Canale (2015)[3] illustrate Simpson's rule in the figure shown below



(a)

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$h = \frac{b-a}{n}$ $n = 4$ $a = x_0$

$I = (b-a)\left(\frac{f(a) + f(b) + 4\sum_{i=1,3} f(x_i) + 2\sum_{i=2} f(x_i)}{3n}\right)$ $b = x_n$

$b = 5$
$a = 0$
$n = 4$

$\frac{b-a}{n} = 1.25$

counter — Points $a + ih$

0 $x = a + ih = 0 + 0(1.25) = 0$

1 $x = a + 1h = 0 + 1(1.25) = 1.25$

2 $x = a + 2h = 0 + 2(1.25) = 2.5$

3 $x = a + 3h = 0 + 3(1.25) = 3.75$

4 $x = a + 4h = 0 + 4(1.25) = 5$

Simpson's 3/8 Rule

- Simpson's 3/8 rule is Newton-Cotes with $n = 3$
- Chapra and Canale (2015)[3] illustrate Simpson's rule in the figure shown below

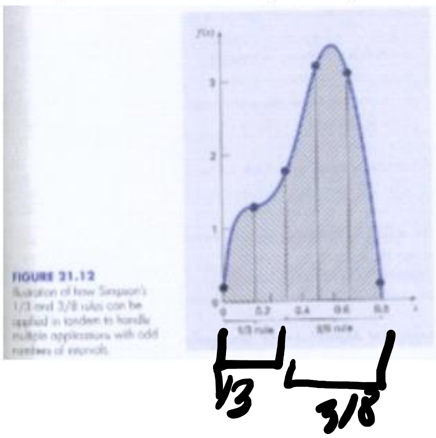

FIGURE 21.12 Illustration of how Simpson's 1/3 and 3/8 rules can be applied in tandem to handle multiple applications with odd numbers of intervals.

n	[illegible] Rule	# of points
1	Trapezoid	2
2	Simpson's 1/3	3
3	Simpson's 3/8	4

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~~Night~~ Test 1

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Test 1 22%

38 → 40 → $-60(.22) = \boxed{-13.2}$ on overall grade

70 → $-30(.22) = -6.6$ points