Section 1

MANE 3351

Lecture 13

Classroom Management

Agenda

- Test 1 not graded
- Homework 3 Assignment
- Lab 6 Assignment
- Numerical Integration

Calendar

Date	Lecture	Lab
10/7	Secant Method, Homework 3	Lab 6
10/9	Trapezoid Rule	Lab 6, continued
10/14	Simpson's Rule, Homework 4	Lab 7
10/16	Romberg Integration	Lab 7, continued
10/21	No Class, Dr. Timmer on ABET Visit	No Class, Dr. Timmer on ABET Visit
10/23	Gaussian Quadrature, Homework 5	Lab 8
10/28	Numerical Differentiation (not on Test 2)	Lab 8, continued
11/4	Numerical Integration (not on Test 2)	Lab 9

Resources

Handouts

- Lecture 13 Slides
- Lecture 13 Marked Slides

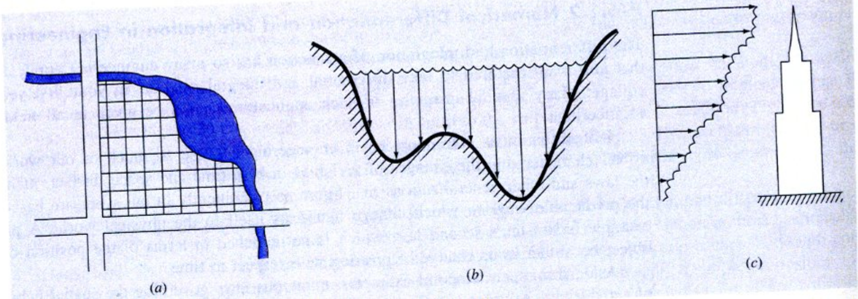
Lecture 13 Content

- Today's topic is numerical integration.
- This is a major new topic after root finding.
- Trapezoid Rule

Introduction to Numerical Integration

- In layman's terms, an integral calculates the area under a curve
- Frequently used in engineering analysis

FIGURE PT6.8
Examples of how integration is used to evaluate areas in engineering applications. (*a*) A surveyor might need to know the area of a field bounded by a meandering stream and two roads. (*b*) A water-resource engineer might need to know the cross-sectional area of a river. (*c*) A structural engineer might need to determine the net force due to a nonuniform wind blowing against the side of a skyscraper.



In electrical field theory, it is proved that the magnetic field induced by a current flowing in a circular loop of wire has intensity

$$H(x) = \frac{4Ir}{r^2 - x^2} \int_0^{\pi/2} \left[1 - \left(\frac{x}{r} \right)^2 \sin^2 \theta \right]^{1/2} d\theta$$

where I is the current, r the radius of the loop, and x the distance from the center to the point where the magnetic intensity is being computed ($0 \leqq x \leqq r$). If I, r, and x are given, we have a nasty integral to evaluate. It is an **elliptic integral** and not expressible in terms of familiar functions. But H *can* be computed precisely by the methods of this chapter. For example, if $I = 15.3$, $r = 120$, and $x = 84$, we find $H = 1.35566\,1135$ accurate to nine decimals.

Figure 2: Another Integration Example

Definitions

Cheney and Kincaid (2004)[a] provide the following definitions

- **Indefinite integral :** $\int x^2 \, dx = \frac{1}{3}x^3 + C$
- **Definite integral:** $\int x^2 \, dx = \frac{8}{3}$

[a]Cheny, W., and Kincaid, D., (2004), *Numerical Mathematics and Computer, 5th edition*

Numerical Integration

Kiusalaas (2013)[a] suggest three major approaches to numerical integration that we will investigate:

1. Newton-Cotes
   a. Trapezoid rule (n=1)
   b. Simpson's rule (n=2)
   c. 3/8 Simpson's rule (n=3)
2. Romberg Integration
3. Gaussian Quadrature

Note: there are many different techniques for numerical integration than the ones listed above

[a] Kiusalaas, J. (2013), *Numerical Methods in Engineering with Python 3*

Newton-Cotes Formulas

Kiusalass (2013)[a] provide the following illustration to explain Newton-Cotes techniques

6.2 Newton-Cotes Formulas

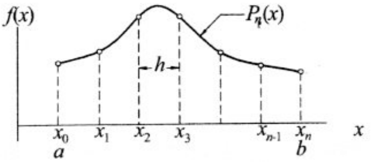


Figure 6.1. Polynomial approximation of $f(x)$.

Figure 3: Newton Cotes Approach

[a]Kiusalaas, J. (2013), *Numerical Methods in Engineering with Python 3*

Trapezoid Rule

Chapra and Canale (2015)[a] provide the figure shown below illustrating the trapezoid rule

FIGURE 21.4
Graphical depiction of the trapezoidal rule.

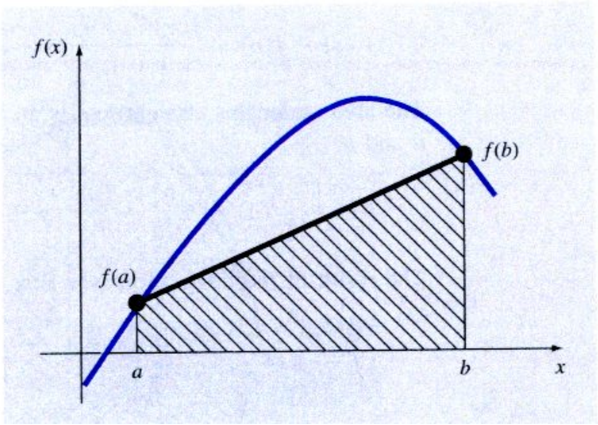


Figure 4: Trapezoid Rule

Trapezoid Rule, continued

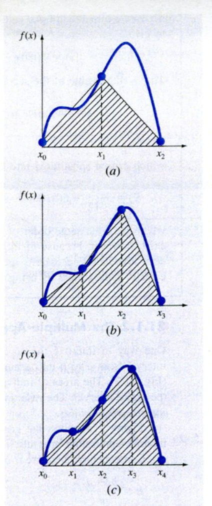
Chapra and Canale (2015)[a] provided the following formulae

- $I = (b - a)\frac{f(a)+f(b)}{2}$
- $E = -\frac{1}{12} f''(\xi)(b - a)^3$

[a] Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

Multiple Applications of the Trapezoid Rule

Typically, the region form a to b is sub-divided into multiple regions and then the Trapezoid Rule for each region is applied. Chapra and Canale (2015)[a] illustrate this concept.



Uniform Spacing

Cheney and Kincaid (2004)[a] the following formula for composite (multiple) applications of the Trapezoid Rule

$$\int_a^b = f(x)dx \approx T(f;P) = h\left\{\sum_{i=1}^{n-1} f\left(x_i\right) + \frac{1}{2}\left[f\left(x_0\right) + f\left(x_n\right)\right]\right\}$$

[a] Cheny, W., and Kincaid, D., (2004), *Numerical Mathematics and Computer, 5th edition*

Pseudo-code

Cheney and Kincaid (2004)[a] provided the following pseudo-code for the composite trapezoid rule

```
program Trapezoid
integer parameter n ← 60
real parameter a ← 0, b ← 1
integer i
real h, sum, x
h ← (b − a)/n
sum ← ½[f(a) + f(b)]
for i = 1 to n − 1 do
    x ← a + ih
    sum ← sum + f(x)
end for
sum ← (sum)h
output sum
end Trapezoid
```

Figure 6: Trapezoid Rule Pseudo-code

Python Code for Multiple Trapezoid Rule Applications

```python
import math
def f(z):
    return (math.exp(-0.5*z**2)/((2.0*math.pi)**0.5))

n=4
a=-5.0
b=0.0
h=(b-a)/n
sum=0.5*(f(a)+f(b))
for i in range(1,n):
    x=a+i*h
    sum=sum+f(x)
sum=sum*h
print("The area is {} for {} sub-intervals".format(sum,n))
```