# Section 1

## MANE 3351

# Lecture 14

## Classroom Management

### Agenda

- Return Test 1
- Simpson's Rule
- Lab 7
- Homework 3 due today
- Homework 4

# Calendar

| Date | Lecture | Lab |
|---|---|---|
| 10/14 | Simpson's Rule, Homework 4 | Lab 7 |
| 10/16 | Romberg Integration | Lab 7, continued |
| 10/21 | No Class, Dr. Timmer on ABET Visit | No Class, Dr. Timmer on ABET Visit |
| 10/23 | Gaussian Quadrature, Homework 5 | Lab 8 |
| 10/28 | Numerical Differentiation (not on Test 2) | Lab 8, continued |
| 11/4 | Numerical Integration (not on Test 2) | Lab 9 |
| 11/6 | Linear Algebra, part 1 (not on Test 2) | Lab 9, continued |
| 11/11 | **Test 2 (Root Finding** | No Lab |

# Resources

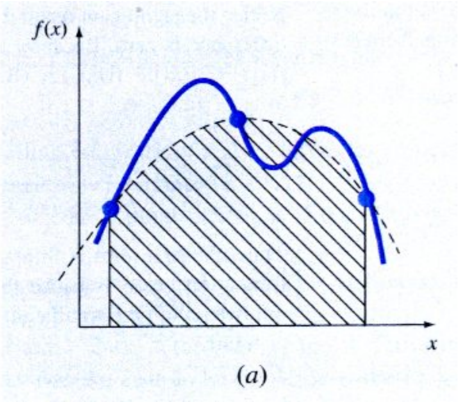
### Handouts

- Lecture 14 Slides
- Lecture 14 Marked Slides

## Lecture 14

Today's topic is Simpson's Rule.

- Simpson's (1/3) rule is Newton-Cotes with $n = 2$
- Chapra and Canale (2015)[a] illustrate Simpson's rule in the figure shown below




[a] Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

## Definition of Simpson's Rule

- Chapra and Canale(2015)[a] provide the following definition of Simpson's rule

$$I = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$$

where $h = (b - a)/2$

- Note that Simpson's rule is of the form

$$I \approx (b - a)\,\frac{f(x_0) + 4f(x_1) + f(x_2)}{6}$$

$$I \approx \text{ width} \times \text{ average height}$$

[a]Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

## Simpson's Rule Error Analysis

- Chapra and Canale (2015)[a] provides the following definition:

$$I = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right] - \frac{1}{90} f^{(4)}(\xi) h^5$$
$$I = \text{Simpson's } 1/3 \text{ approximation} - \text{ Truncation error}$$

[a] Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

### Multiple Applications of Simpson's Rule

- The number of segments must be even
- The formula is

$$I \approx (b-a)\,\frac{f(x_0) + 4\sum_{i=1,3,5}^{n-1} f(x_i) + 2\sum_{j=2,4,6}^{n-2} f(x_j) + f(x_n)}{3n}$$

where $h = \frac{b-a}{n}$

# Pseudo-code: Simpson's 1/3 Rule

- CodeSansar[a] provides the following pseudo-code

Simpson's 1/3 Rule Algorithm

```
1. Start

2. Define function f(x)

3. Read lower limit of integration, upper limit of
   integration and number of sub interval

4. Calcultae: step size = (upper limit - lower limit)/number of sub interval

5. Set: integration value = f(lower limit) + f(upper limit)

6. Set: i = 1

7. If i >= number of sub interval then goto step 11

8. Calculate: k = lower limit + i * h

9. If i mod 2 =0 then
     Integration value = Integration Value + 2* f(k)
   Otherwise
     Integration Value = Integration Value + 4 * f(k)
   End If

10. Increment i by 1 i.e. i = i+1 and go to step 7

11. Calculate: Integration value = Integration value * step size/3

12. Display Integration value as required answer

13. Stop
```

[a] https://www.codesansar.com/numerical-methods/integration-simpson-1-3-method-algorithm.htm

# Python Code for Multiple Simpson's 1/3 Rule

```
import math
def f(z):
    return (math.exp(-0.5*z**2)/((2.0*math.pi)**0.5))

n=10
a=-5.0
b=0.0
h=(b-a)/n
sum=f(a)+f(b)
for i in range(1,n):
   # print(i)
    k=a+i*h
    if i%2==0:
        #print("even number")
        sum=sum+2.0*f(k)
    else:
        #print("odd number")
```

# Simpson's 3/8 Rule

- Simpson's 3/8 rule is Newton-Cotes with $n = 3$
- Chapra and Canale (2015)[a] illustrate Simpson's rule in the figure shown below

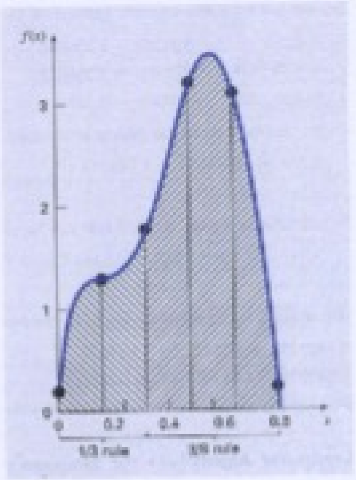



**FIGURE 21.12**
Illustration of how Simpson's 1/3 and 3/8 rules can be applied in tandem to handle multiple applications with odd numbers of intervals.

[a]Chapra, S., and Canale, R., (2015). *Numerical Methods for Engineers, 7th edition*

## Definition of Simpson's 3/8 Rule

- Chapra and Canale(2015)[a] provide the following definition of Simpson's rule

$$I = \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right]$$

where $h = (b - a)/3$

- Note that Simpson's 3/8 rule is of the form

$$I \approx (b - a)\,\frac{f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)}{8}$$

$$I \approx \text{ width} \times \text{ average height}$$

[a] Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

### Truncation Error of Simpson's 3/8 Rule

$$E_t = -\frac{3}{80} h^5 f^{(4)}(\xi)$$
$$= -\frac{(b-a)^5}{6480} f^{(4)}(\xi)$$

## Classroom Coding: One Interval of Simpson's 3/8 Rule

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