Section 1

MANE 3351

Subsection 1

Lecture 16

Classroom Management

Agenda

- Gaussian Quadrature
- Homework 5 (due October - no late work)
- Lab session postponed

Subsection 2

Resources

Handouts

- Lecture 16 slides
- Lecture 16 slides marked
- Abscissas and Weights for Gaussian Integration

Calendar

Lecture/Lab	Date	Topic
10/21	No Class, Dr. Timmer on ABET Visit	No Class, Dr. Timmer on ABET Visit
10/23	Gaussian Quadrature, Homework 5	Lab 8 postponed
10/28	Numerical Differentiation (not on Test 2)	Lab 8
11/4	Numerical Integration (not on Test 2)	Lab 8, continued
11/6	Linear Algebra, part 1 (not on Test 2)	Lab 9
11/11	**Test 2 (Root Finding and Numerical Integration)**	No Lab

Lecture 16

Today's topic is Gaussian Quadrature

- $\int_{-1}^{1} f(x)dx \approx \sum_{i=1}^{n} w_i f(x_i) + R_n$
- x_i is the abscissa is the i-th zero of $P_n(x)$ (Legendre polynomial)
- w_i is the weight associate with x_i
- $R_n = \frac{2^{2n+1}(n!)^4}{(2n+1)[(2n)!]^3} f^{(2n)}(\xi)$
- Abscissas and weights are commonly tabled [1]

[1] Abramowitz, M., and Stegun, I. (1972), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*

Python Code, part 1

```
# Part 1, original formula in range (-1,1)
#
import math
import numpy as np
def f(z):
    return (math.exp(-0.5*z**2)/((2.0*math.pi)**0.5))
#
# 2 point Gaussian Quadrature
x2=[-0.577350269189626,0.577350269189626]
w2=[1.0,1.0]
#
i2=0.0
for i in range(0,len(x2)):
    i2=i2+w2[i]*f(x2[i])
print("the area using 2-point gaussian quadrature is {}".forma
#
# 5 point Gaussian Quadrature
```

Gauss' Formula, Arbitrary Interval

- $\int_a^b f(y)dy \approx \frac{b-a}{2} \sum_{i=1}^{n} w_i f(y_i) + R_n$
- $y_i = \left(\frac{b-a}{2}\right) x_i + \left(\frac{b+a}{2}\right)$
- $R_n = \frac{(b-a)^{2n+1}(n!)^4}{(2n+1)[(2n)!]^3} f^{(2n)}(\xi)$
- Note that x_i and w_i are the points and weights of Gaussian quadrature

Python Code, part 2

```
# Part 2
#
print("Gaussian Quadrature, arbitrary interval")
a=-5.0
b=0.0
#
# redo 2 point gaussian
i2=0.0
for i in range(0,len(x2)):
    y=((b-a)/2.0)*x2[i]+(b+a)/2.0
    i2=i2+w2[i]*f(y)
i2=((b-a)/2.0)*i2
print("the area using 2-point gaussian quadrature is {}".forma
#
# redoc 5 point gaussian
i5=0.0
for i in range(0,len(x5)):
```

ChatGPT program

- ChatGPT can be a useful tool for developing Python code.
- It should not replace your knowledge and expertise
- ChatGPT can be wrong